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Atmospheric static stability in respect to static energy changes

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This theoretical study deals only with the static part of the total energy and tends to explore the functional relationship between its changes and atmospheric static stability. It is shown that a dimensionless energy number may be defined in terms of isentropic analysis to be the function of static stability with recognizable physical meaning. Its mathematical properties indicate that this particular dependence may be described by a bilinear fractional function taking a form of an equilateral hyperbola with asymptotes parallel to the coordinate axes. Its unstable branch corresponds to the higher rate of change of static energy and lies in the first quadrant, while in the third one its lower rate is related to the stable branch. Asymptotic solution of considered theoretical problem implies that the transition from stable to unstable branch may appear only by an infinite jump and holds an intriguing resemblance with either the quantum transition between two energy shells by the absorption or emission of energy or the transition from stable to unstable regime of the general atmospheric circulation.

Atmosferska statička stabilnost i promjene statičke energije

U radu se razmatra samo statički dio ukupne energije i nastoji istražiti funkcijski odnos njezinih promjena i atmosferske statičke stabilnosti. Pokazuje se da je primjenom izentropskih jednadžbi moguće definirati bezdimenzijski energetski broj prepoznatljivog fizikalnog značenja u zavisnosti o statičkoj stabilnosti, pogodan za objektivni opis i interpretaciju ove funkijske veze. Njegova matematička svojstva ukazuju da je razmatrani odnos razlomljena funkcija, te da ima oblik jednakostrane hiperbole, čije su asimptote paralelne koordinatnim osima. Njezinoj nestabilnoj grani odgovaraju veći iznosi i brže promjene statičke energije, dok su na stabilnoj ovi iznosi manji, a promjene sporije. Asimptotsko rješenje razmatranog teoretskog problema upućuje na zaključak da je prijelaz iz stabilnog u nestabilno stanje i obratno moguć samo uz skokovitu promjenu energetskog sadržaja česti zraka dodavanjem ili oduzimanjem energije. Pritom je evidentna sličnost kako s kvantnim prijelazom elektrona između susjednih energetskih ljuski uvjetovanim absorpcijom ili emisijom energije, tako i s prijelazom iz stabilnog u nestabilni režim globalne cirkulacije atmosfere.

1. Introduction

The great importance of the static stability parameters and their very close relation to the conversion of atmospheric energy have for long been recognised by many authors. Nevertheless, it was examined and theoretically explored by most of them mainly in the frame of the macroscale motions or by considering the general circulation dynamics. This is one of the well-known and fundamental subjects in the dynamics of the atmospheric systems, where the question which has to be answered is not only what happens to the energy involved, but also how and when its changes are taking place. Lorenz (1955, 1960) pointed out some pronounced consequences of this relation. The first one is related to the global scale indicating the existence of the net long-term conversion of potential and internal energy into the kinetic energy by isentropic processes. As a result there is a net long-term increase in the overall static stability that must necessarily be balanced by an appropriate net decrease of stability caused by diabatic effects. On the other hand, the processes on the synoptic scale, such as a cyclone development, may produce a considerable increase of static stability as a consequence of the kinetic-energy-producing circulation. This includes rising of warmer air and simultaneous sinking of colder and dense air which tends to be placed underneath. Such a growth of static stability inhibits further development of a cyclone. According to Lorenz, the consequences on the local scale are nearly the same. This means that any convective process tends to be inhibited when increasing static stability tends to a given threshold value. Concerning the general circulation of the atmosphere Lorenz (1984) stressed that instability may be considered to be the most important property of chaotic dynamical systems and the cause of atmospheric irregularity.

The considered feature of static stability was implemented in the theory of baroclinic flow by Charney (1947) and Eady (1950) to define an appropriate criterion for the growth of a small preturbation superimposed on a zonal current. The former is based on the same fact that the high static stability values represent a mechanism to prevent any greater perturbation growth.

The objective of this work is a theoretical study of the functional relationship between atmospheric static stability and static energy changes in terms of isentropic analysis. A great deal of attention was paid to an objectively derived description of this particular connection that could bring more light into its mathematical properties and physical consequences. The temperature lapse rate is chosen as a pronounced local characteristic of the atmosphere to describe static stability at a given point, while only the static part of the total atmospheric energy consisting of the internal and gravitational potential energy is taken into account. As shown by Margules (1903) for the case of thermally isolated system under quasistatic conditions, these two types of energy are proportional to each other. Their combination was termed the total potential energy (*TPE*) and may be treated as a single form of energy to simplify the energetics problem. Following the notation given by Bluestein (1992, p. 272–273) and the similar approaches in deriving the *TPE* equation by Brunt (1952, p. 283) and by Godske et al. (1957, p. 456) an important expression may be obtained in the form:

$$TPE = I + PE = \int (c_v T + \Phi)\,\mathrm{d}m + \mathrm{const.} = \int c_p\, T\,\mathrm{d}m + \mathrm{const.}\,, \qquad (1)$$

where $\mathrm{d}m = \rho\,\mathrm{d}z$ represents the element of mass of a volume per unit horizontal area, c_v and c_p denote specific heat at constant volume and constant pressure, is the density and T the absolute temperature. Having the physical dimension of specific energy, the parameters $c_v T$, $c_p T$ and Φ correspond to the internal energy, enthalpy and gravitational potential energy per unit mass, respectively.

The equation (1) states that the specific enthalpy $c_p T$ itself may be employed to represent the total potential energy usually defined by the sum $c_v T + \Phi$. Accordingly, this paper deals with the static energy and its changes by investigating the behaviour of the specific enthalpy against a number of thermodynamic and stability parameters in the isentropic coordinate system. Due to its obvious similarity with defining relation of the Montgomery potential function M in which so-defined static energy is implicitly contained and combined with geopotential height as

$$M = c_p T + \Phi \equiv \varepsilon\Theta + \Phi \qquad (2)$$

the function M has been used in deriving the appropriate expressions describing the relationship of static energy changes and static stability. Its hydrostatic change with respect to the potential temperature Θ is defined by the isentropic hydrostatic equation of the form

$$\left(\frac{\partial M}{\partial \Theta}\right)_h = \varepsilon = \frac{c_p T}{\Theta} = c_p \left(\frac{p}{p_0}\right)^{\kappa}, \qquad \kappa = \frac{R_d}{c_p}, \qquad (3)$$

where ε denotes the Exner function that is often called *modified pressure*, p is the pressure, R_d the gas constant for dry air, while subscript h refers to hydrostatic conditions. By combining the equation (3) with the first two laws of thermodynamics it can be easily shown that the changes of both the static energy and the Montgomery potential function are clearly related to the specific entropy s and the heat Q either added to or removed from the system. This follows from identities

$$T\,\mathrm{d}s \equiv c_p T\,\mathrm{d}(\ln\Theta) = c_p\,\mathrm{d}T - \frac{\mathrm{d}p}{\rho} = \mathrm{d}(c_p T + \Phi) = \mathrm{d}M \equiv \delta Q, \tag{4}$$

where the lowercase letter δ denotes inexact differential of the heat function.

This study indicates that a dimensionless energy number may be defined to be the function of static stability and correlated to the other thermodynamic parameters. The latter enables the physical interpretation of its mathematical properties showing that this functional dependence takes the form of an equilateral hyperbola with asymptotes separating its stable and unstable branches. Asymptotic behaviour of considered relationship implies that the transition from stable to unstable branch may happen only by an infinite jump forced either by adding the energy to the system or by its releasing. Some theoretical remarks of this have been reported by Glasnović (1993) and applied to the specific problems by Glasnović et al. (1994).

2. Dimensionless number describing static energy changes

Taking the Montgomery potential function M to examine the behaviour of the specific static energy c_pT, a dimensionless quantity may be derived in terms of the conventional isentropic analysis to describe its changes against the change of any thermodynamic parameter. Differentiation of its defining expression (2) with respect to the geopotential height Φ and potential temperature Θ leads to an appropriate relationship of the form

$$\frac{\partial M}{\partial \Phi} = \frac{\partial M/\partial\Theta}{\partial\Phi/\partial\Theta} = 1 + \frac{\varepsilon\left(1 + \dfrac{\partial \ln\varepsilon}{\partial \ln\Theta}\right)}{\partial\Phi/\partial\Theta}\,. \tag{5}$$

As suggested by Glasnović (1993) the nominator of the second term on the right side in equation (5) can be used as a dimensionless parameter to describe the static energy changes with respect to the change of the potential temperature Θ and either the pressure p or the Exner function ε. It was termed dimensionless energy number $\hat{E}$ and its defining relations were expressed as

$$\hat{E}(\varepsilon,\Theta) \equiv 1 + \frac{\partial \ln\varepsilon}{\partial \ln\Theta} = \frac{1}{\varepsilon}\frac{\partial \varepsilon\Theta}{\partial\Theta} \equiv 1 + \kappa\frac{\partial \ln p}{\partial \ln\Theta} = \hat{E}(p,\Theta). \tag{6}$$

On the other hand, the alternative, but identically the same relations may be derived by logarithmic differentiation of temperature with respect to the pressure and potential temperature in the forms

$$\hat{E}\{T(p,\Theta)\} \equiv \frac{\partial \ln T}{\partial \ln \Theta} = 1 + \kappa \frac{\partial \ln p}{\partial \ln \Theta} \equiv 1 + \frac{\partial \ln \varepsilon}{\partial \ln \Theta} = \frac{1}{\varepsilon} \frac{\partial c_p T}{\partial \Theta} \equiv \frac{\partial c_p T / \partial \Theta}{(\partial M / \partial \Theta)_h} \qquad (7)$$

indicating that $\hat{E}$ compares the local changes of the specific enthalpy $c_p T$ with potential temperature Θ to the hydrostatic change of the Montgomery potential M defined according to Eq. (3) by the Exner function ε.

The qualitative consideration of the equations above refers to the special and physically recognisable states of reference. The most important one is shown in Fig. 1 and corresponds to the state of stable thermodynamic equilibrium under hydrostatic conditions at which the thermodynamic potentials (see Fermi, 1936) take the minimum values and the surfaces of constant M and Θ coincide with level surfaces $\Phi = \text{const}$. The pressure at considered point is hydrostatic one due solely to the weight of the fluid above. The tangential forces vanish, while the normal ones are equal. This state is horizontally homogeneous and the fluid elements are either in a state of rest or subjected to the stable oscillations with respect to its equilibrium position. Consequently, $\partial M / \partial \Phi$ is equal to one so that the second term defining $\hat{E}$ on the right side of (5) is equal to zero.

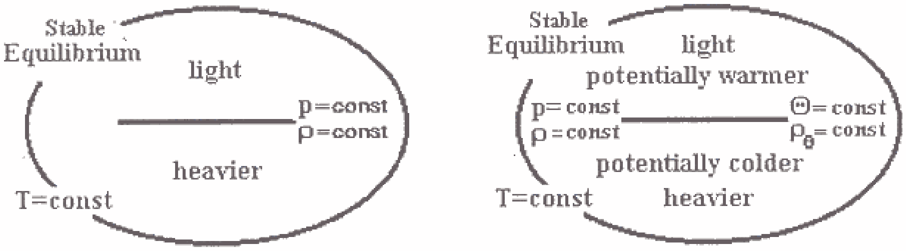

Figure 1. Stable thermodynamic equilibrium as theoretical state of reference under hydrostatic conditions. The left part illustrates the geometric distribution of pressure and density in a compressible fluid at rest, while the state on the right corresponds to the isothermal conditional barotropy.

The mathematical formulation of the coincidence condition stated above may be obtained by determining the relationship among the lapse rate of the Montgomery potential function and its horizontal gradients along both of the level surfaces and the surfaces of constant static energy. Denoting the corresponding gradients by the subscripts Φ and M, the gradient of the static energy $c_p T$ on the constant Montgomery potential surfaces may be expressed as

$$\left(\frac{\partial c_p T}{\partial x,y}\right)_M = \frac{\partial c_p T}{\partial \Phi}\left(\frac{\partial \Phi}{\partial x,y}\right)_M + \left(\frac{\partial c_p T}{\partial x,y}\right)_\Phi . \qquad (8)$$

Differentiations of the original expression for Montgomery potential function (2) with respect to Φ as well as to x or y, but holding Φ and M constant, leads to the relations

$$\frac{\partial M}{\partial \Phi} = 1 + \frac{\partial c_p T}{\partial \Phi}, \quad \left(\frac{\partial c_p T}{\partial x,y}\right)_{\Phi} = \left(\frac{\partial M}{\partial x,y}\right)_{\Phi}, \quad \left(\frac{\partial c_p T}{\partial x,y}\right)_{M} = -\left(\frac{\partial \Phi}{\partial x,y}\right)_{M}, \tag{9}$$

Substituting (9) into the differential equation (8) it may be rearranged and reduced to the form

$$\frac{\partial M}{\partial \Phi}\left(\frac{\partial \Phi}{\partial x,y}\right)_{M} = -\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} \tag{10}$$

giving the basis for detailed physical interpretation. This indicates two special equilibrium states of reference. The first one includes the coincidence of surfaces of constant Montgomery potential and constant potential temperature with level surfaces in the space of constant static energy when

$$\frac{\partial M}{\partial \Phi} = 1, \quad \left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = \left(\frac{\partial \Phi}{\partial x,y}\right)_{\Phi} \equiv 0, \quad \left(\frac{\partial c_p T}{\partial x,y}\right)_{\Phi} = 0, \quad \frac{\partial c_p T}{\partial \Phi} = 0. \tag{11}$$

Combining the isentropic hydrostatic equation (3) with (11) and taking the differential expression (10) into account, the identity may be derived in the form

$$\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = \varepsilon\left(\frac{\partial \Theta}{\partial x,y}\right)_{\Phi} = -\left(\frac{\partial \Phi}{\partial x,y}\right)_{M} \equiv 0 \tag{12}$$

that asserts the above coincidence condition. Since the specific enthalpy does also have the meaning of thermodynamic potential, the criterion to examine the type of equilibrium may be applied requiring that in a stable state thermodynamic potentials must take the minimum absolute value. According to (7), (11) and (12), the first derivative of $c_p T$ vanishes so that

$$\hat{E} = 0, \qquad \varepsilon\Theta = c_p T = \text{const.}, \quad \varepsilon\frac{\partial \Theta}{\partial \Phi} = 1 \tag{13}$$

and the above requirement is entirely satisfied indicating that this state of constant static energy, i.e. of constant *TPE*, corresponds to the stable thermodynamic equilibrium under hydrostatic conditions. It can be also formulated by the statement that there is only one reference state at rest that possesses the least static energy, at which the surfaces of constant pressure, potential temperature, Montogomery potential and density are all horizontal, and the potential temperature always increases with elevation.

Another theoretical state of reference is related to the neutral equilibrium at the dry-adiabatic temperature lapse rate and may exist in an environment of the constant Montgomery potential and constant potential temperature. Such a stratification is unstable and could be established only in a fluid absolutely at rest, when the constant static energy surfaces are horizontal and coinciding with level surfaces of constant geopotential height. Under such circumstances the following relationships are valid

$$\frac{\partial c_p T}{\partial \Phi} = -1, \quad \left(\frac{\partial c_p T}{\partial x,y}\right)_{\Phi} = -\left(\frac{\partial \Phi}{\partial x,y}\right)_{\Phi} \equiv 0, \quad \left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = 0, \quad \frac{\partial M}{\partial \Phi} = 0. \qquad (14)$$

Substituting the isentropic hydrostatic equation (3) into (14), it may be expressed in the forms

$$\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = \varepsilon\left(\frac{\partial \Theta}{\partial x,y}\right)_{\Phi} \equiv 0, \quad \frac{\partial M}{\partial \Phi} = \varepsilon\,\frac{\partial \Theta}{\partial \Phi} \equiv 0 \qquad (15)$$

that refer to the state at which the potential temperature does not change either in the vertical direction or along the level surface of constant geopotential. Taking the dimensionless number to be the function of both the static energy and the Montgomery potential that is expressed as

$$\hat{E}(c_p T, M) = 1 - \frac{\partial \Phi}{\partial M} = \frac{\partial c_p T}{\partial M}, \qquad (16)$$

it follows that, under conditions described by (14) and (15), $\hat{E}$ is related to an adiabatic environment and takes an infinite value, *i. e.*, an absolute maximum of static energy.

In a more general case illustrated in Fig. 2, when the tangential forces do not vanish and the state is not horizontally homogeneous, the pressure is not necessarily hydrostatic and there is a displacement of surfaces from their equilibrium position. Dimensionless number $\hat{E}$ is not zero and $\partial M/\partial\Phi$ differs from the unit. It can be formulated by the relationships of the form

$$\frac{\partial M}{\partial \Phi} \neq 1, \quad \frac{\partial c_p T}{\partial \Phi} \neq 0, \quad \left(\frac{\partial \Phi}{\partial x,y}\right)_{M} \neq -\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} \neq 0, \qquad (17)$$

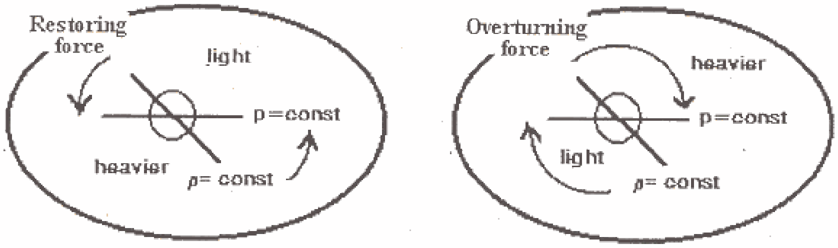

Figure 2. Displacements from hydrostatic equilibrium by the tilt of density surfaces in the case of compressible fluid initially at rest (according to Turner, 1979). The left part refers to the oscillations about the equilibrium position and indicates the influence of the restoring force at stable density distribution. On the right part the density distribution is unstable and an overturning force leads necessarily to the convective motion.

where

$$\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = \varepsilon\left(\frac{\partial \Theta}{\partial x,y}\right)_{\Phi} + \Theta\left(\frac{\partial \varepsilon}{\partial x,y}\right)_{\Phi} \neq 0, \tag{18}$$

showing that the surfaces of constant properties are tilted with respect to the level surfaces and that their mutual coincidence is also violated. The second term on the right side of equation (18) is proportional to the horizontal pressure gradient and represents its *nonhydrostatic contribution* to the slope of the constant Montgomery potential surfaces against the horizontal as well as with respect to the isentropic surfaces. When this term vanishes the hydrostatic conditions are established and described by the relations

$$\left(\frac{\partial M}{\partial x,y}\right)_{\Phi} = \varepsilon\left(\frac{\partial \Theta}{\partial x,y}\right)_{\Phi} \neq 0, \quad \left(\frac{\partial \varepsilon}{\partial x,y}\right)_{\Phi} = 0, \tag{19}$$

stating that the isobaric surfaces are horizontal, while the surfaces of constant M and Θ are not. Nevertheless, they mutually coincide at an angle against level surfaces.

Concerning the theoretical state defined at the value $\hat{E} = 1$ it will be shown below that it corresponds to the asymptotic solution of considered theoretical problem and therefore it can never be established in reality. According to the relations (6) and (7), it would be required that the potential temperature does not depend on the pressure, implying a state of infinite static stability or instability.

The above discussion refers to some important features of considered dimensionless quantity that may have not only theoretical, but also great practical meaning. It can be correlated to the slope of the surfaces of constant physical properties against the horizontal and used as a measure of deviation from the stable thermodynamic equilibrium existing under hydrostatic conditions.

3. Dimensionless number $\hat{E}$ as a function of static stability

Another important meaning that may be assigned to considered dimensionless parameter is characterized by its correlation with atmospheric stability. Choosing the temperature lapse rate γ for local stability description, it can be easily proved that functional dependence $\hat{E}(\gamma)$ and its inverse variation $\gamma(\hat{E})$ take the forms

$$\hat{E} = \hat{E}(\gamma) = \frac{\gamma}{\gamma - \Gamma_d}, \qquad \gamma(\hat{E}) = \Gamma_d \frac{\hat{E}}{\hat{E} - 1}, \tag{20}$$

where in their deriving either the eqation (6) or its alternative (7) have been utilized in combination with the temperature, dry-adiabatic and potential temperature lapse rates usually expressed as

$$\gamma = -\frac{\partial T}{\partial z}, \qquad \Gamma_d = \frac{g}{c_p}, \qquad \frac{\partial \Theta}{\partial z} = \frac{\Theta}{T}(\Gamma_d - \gamma). \tag{21}$$

The equations (20) indicate that the considered mutually inverse functions are both fractional and symetric with respect to the bisection of the first and third quadrants. As shown in Fig. 3, their behaviour is completely described by equilateral hyperbola with asymptotes $\hat{E} = 1$ and $\gamma = \Gamma_d$ parallel to coordinate axes. The main feature of both graphs are their physically quite recognizable branches of opposite stability and different rate of change of static energy implying nonlinearity of these relationships. Their asymptotes necessarily have the special meaning indicating characteristic tresholds or the critical values for transition from one branch to another.

Investigating the mathematical properties of functional relationship $\hat{E}(\gamma)$ two special states of interest may be determined. Passing to the limit, when $\gamma \to 0$, refers to a state

$$\lim_{\gamma \to 0} \hat{E}(\gamma) = \lim_{\gamma \to 0} \frac{\gamma}{\gamma - \Gamma_d} = 0 \tag{22}$$

at which the thermodynamic potentials have the minimum absolute value indicating the stable thermodynamic equilibrium of an isothermal and compressible fluid element at rest under hydrostatic conditions. This element is thermally balanced with its environment so that the principle of coincidence

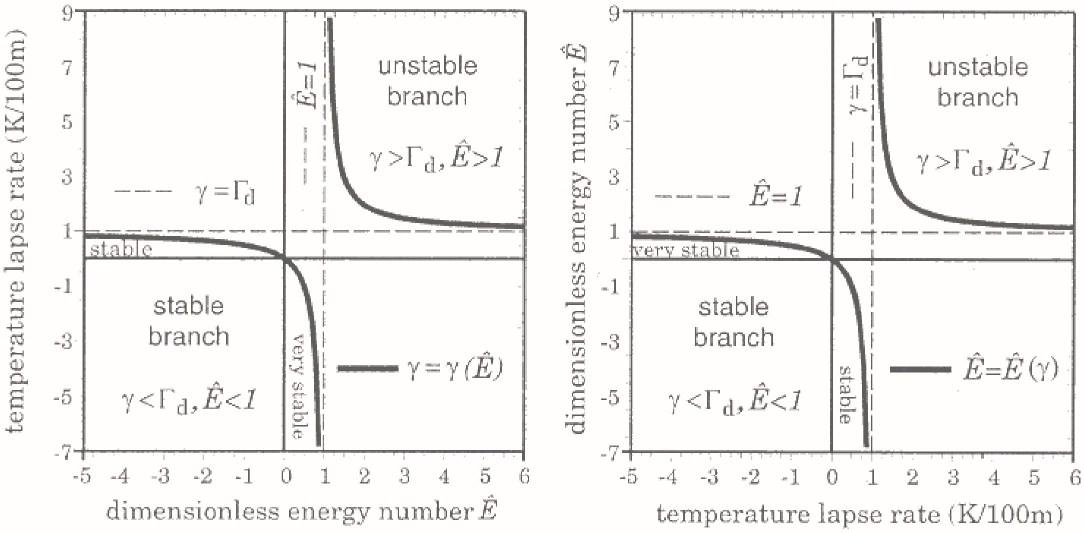

Figure 3. The graphs of two mutually inverse functions $\gamma(\hat{E})$ and $\hat{E}(\gamma)$ which are symetric with respect to the bisection of the first and third quadrants. Both are expressed by the bilinear fractional function taking the form of equilateral hyperbola with assymptotes separating their branches of quite opposite stability.

of surfaces is entirely satisfied. Besides, the former is the only state at which both parameters take the same numerical value. In terms of the chaos theory (see Tsonis and Elsner, 1989), in the so-called *space state* the point $\hat{E}=0, \gamma=0$ corresponds to a *point attractor* to which any oscillating air particle will converge regardless of its initial position. As argued in the previous section, it contains the minimal absolute value of static energy and may be taken as a reference state to measure deviations in static stability and static energy contents.

Another state of stability corresponds to the unstable or neutral hydrostatic equilibrium of an incompressible fluid element at rest characterized by dry-adiabatic lapse rate. At this point of its argument γ, the function $\hat{E}(\gamma)$ is not defined, so that, when $\gamma \to \Gamma_d$, it follows:

$$\lim_{\gamma \to \Gamma_d} \hat{E}(\gamma) = \lim_{\gamma \to \Gamma_d} \frac{\gamma}{\gamma - \Gamma_d} = \pm\infty \tag{23}$$

indicating one of the asymptotic solutions of the considered problem for $\gamma = \Gamma_d$ at which the rate of change of static energy would become extremely large. As described by relationships (14)–(16), this is related to an adiabatic space as well as to the space of constant Montgomery potential function. The constant static energy surfaces are then horizontal and their vertical distribution is proportional to the dry-adiabatic lapse rate of temperature.

The same qualitative conclusions may be drawn by investigating the limiting behaviour of the functional relationship $\gamma(\hat{E})$. It can be easily proved that $\gamma(\hat{E})$ possesses an *infinite discontinuity* when its argument $\hat{E}$ tends to one. The passage to the limit indicates

$$\lim_{\substack{\hat{E}\to 1\\ \hat{E}<1}} \gamma(\hat{E}) = -\infty, \qquad \lim_{\substack{\hat{E}\to 1\\ \hat{E}>1}} \gamma(\hat{E}) = +\infty \tag{24}$$

and refers to another asymptotic solution at $\hat{E} = 1$ of the extreme static stability that cannot be met in reality. It states that, as approaches +1 assuming only negative (positive) values, the considered function tends to $-\infty$ $(+\infty)$ and its absolute value becomes of infinitely large magnitude. On the other hand, the function $\gamma(\hat{E})$ tends to one and the same value Γ_d when its argument either increases or decreases unlimitedly, and analogous behaviour can be found for its inverse $\hat{E}(\gamma)$. It may be expressed by passing to the limits

$$\lim_{\hat{E}\to\pm\infty}\gamma(\hat{E}) = \Gamma_d \lim_{\hat{E}\to\pm\infty}\frac{1}{1-\dfrac{1}{\hat{E}}} = \Gamma_d, \qquad \lim_{\gamma\to\pm\infty}\hat{E}(\gamma) = \lim_{\gamma\to\pm\infty}\frac{1}{1-\dfrac{\Gamma_d}{\gamma}} = 1 \tag{25}$$

that determine the asymptotes of equilateral hyperbolas.

The striking feature of both considered functions is their mutual nonlinearity that can be clearly recognized by their graphs. The nonlinear behaviour is occuring in the same way on both the lower branch that is stable, at $\gamma < \Gamma_d$ and $\hat{E} < 1$, and the upper branch that is unstable, for $\gamma > \Gamma_d$ and $\hat{E} > 1$. The stable vertex on the graphs of both functional relationships corresponds to the state of thermodynamic equilibrium at which γ and $\hat{E}$ simultaneuosly become zero. The vertex on the unstable branch that is characterized by $\gamma = 2\,\Gamma_d$ and $\hat{E} = 2$ has no special meaning.

Regarding the behaviour of γ and $\hat{E}$ with respect to the third quantity, such as the lapse rate of the Montgomery potential, an additional relationship may be derived as

$$\frac{\partial M}{\partial \Phi} = 1 - \frac{\gamma}{\Gamma_d} = \frac{1}{1-\hat{E}}, \tag{26}$$

indicating a linear functional relationship between the considered lapse rates, but their nonlinearity against the static energy changes. It can be realized as an objective indication that atmospheric nonlinearities are primarily caused by nonlinear behaviour of the static energy changes.

4. Conclusions

The most important result of this theoretical study corresponds to the asymptotic solutions of the functional relationships $\hat{E}(\gamma)$ and $\gamma(\hat{E})$. For the latter case, this indicates that, when the argument $\hat{E}$ passes from the values smaller than 1 to those greater than 1, the function $\gamma(\hat{E})$ *makes an infinite jump* from the stable to the unstable branch of hyperbola. The same behaviour can be recognized for its inverse variation $\hat{E}(\gamma)$ when the temperature lapse rate is taken to be the independent variable. The critical threshold value for the jump is found to be the asymptotic value of the dimensionless energy number for $\hat{E} = 1$.

Physical interpretation of their mathematical properties indicates that they can be taken as objectively derived evidence to prove that *any essential change of thermodynamic state of the fluid element implying the change of its stability type may occur solely by the jump of its energy contents.* Since the latter should be highly related to the addition or release of large amounts of energy, it can be understood as a mechanism capable to overcome the influence of very high static stability that usually prevents any further development of atmospheric systems. Concening atmospheric phenomena on various scales of motion it follows that the static energy jump may primarily be encountered as an extremely local process. This in interesting analogy with the quantum theory of the atom implying an intriguing resemblance with the quantum transition between two energy shells by the absorption or emission of energy. Another analogous example, that was pointed out by Lorenz (1984), is related to the general circulation of the atmosphere and the irregularity of the chaotic dynamical systems, where both the instability and aperiodicity play an important role in the transition from asymmetric to symmetric regime, *i. e.*, from stable to unstable general circulation.

The consideration of bilinear fractional function $\hat{E}$ in combination with the local stability defined by the temperature lapse rate γ gives the physical basis to establish the following criterion

$\hat{E} > 1$,	unstable state at higher rate of change of static energy
$\hat{E} < 1$,	stable state at lower rate of change of static energy
$0 < \hat{E} < 1$,	very stable state / temperature inversion for $\gamma < 0$

indicating its usefulness and applicability in diagnostics of special atmospheric phenomena. This is related to its property to be a measure of deviation from the stable hydrostatic equilibrium of an air particle at rest. It enables to get more light into the general atmospheric states that are more or less nonhydrostatic and to identify the striking features of the vertical atmospheric structure such as either the superadiabatic layers or temperature inversions of frontal type.

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