EVALUATION OF THE VEHICLE REPAIR SUBSYSTEM WITH MARKOV MODELS

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Abstract

Analysis and evaluation of repair subsystem in the transport system is extremely important. It has a direct impact on the quality of performed tasks and the readiness level of surveyed organization. It also enables to identify factors that have a negative impact on the repair process, its duration and quality. These include e.g. waiting times for spare parts, for specialists (mechanics) or free repair stations (channels), as well as availability of equipment or tools necessary for implementation of renewal procedures. The level of training, knowledge and experience of mechanics, which can be key to the success of planned repairs, is also extremely important.

This article presents an analysis of the example of repair subsystem. The Markov models were used for diagnostics, which enabled to determine the probability limit values of duration of the operating states distinguished in this subsystem and, on this basis, to determine the imperfections of implemented processes and possibility of their improvement.

Key words: Markov models, vehicle readiness, repair subsystem

1. INTRODUCTION

Complex systems of operations require a proper management strategy not only in terms of use and tasks carried out, but also in terms of planned maintenance and repairs. Regular inspection of the technical condition of objects is a form of support in this respect. One of the ways to effectively counteract the destructive processes that accompany each and every machine from the moment of its manufacturing until its decommissioning is proper scheduling of preventive replacements and improvement of diagnostic methods (Żółtowski, 2012).

There are many strategies of object operation within the company, with the most common ones including (Żółtowski, 2012):

- according to reliability,
- according to economic efficiency,
- according to the amount of work done,
- according to technical condition.

The most desirable approach is preventive maintenance that anticipates the inoperable state of a technical object and prevents failures from occurring. It allows to avoid additional costs, but at the same time requires systematic monitoring and diagnostics to precisely determine the moment preceding the failure, which ultimately leads to sufficiently long operation of a given element.

The choice of the right strategy of object operation is, however, not enough. It is also important to regularly control the processes carried out within its framework. In the transport company presented in the article an analysis of the readiness of the owned vehicles was carried out, which indicated that the vehicles spend more than 20% of the total time of operation in the repair subsystem. It resulted in the decision to carry out a detailed analysis of the repair activities carried out in order to answer the question whether it is justified to devote 1/5 of the time to repairs. An additional objective of the article was to present a mathematical method enabling such a study. Markov processes were used.

Examples of application of Markov models are common in literature, especially in the area of reliability and readiness of both individual elements (Alvarez & Lane, 2016) and complex systems (Hu et al., 2017; Pilch, 2017). Studies are carried out for discrete time, in which binary implementations of operating states are possible, as e.g. in (Eryilmaz, 2016; Levitin et al. 2017). Multistate combinations are also analyzed, both in discrete (Troaes, 2015) and continuous time (Iscioglu & Kocak, 2019; Liu et al. 2016; Liu & Zio, 2017).

The area which is most frequently explored is the wear of individual devices (elements) or assemblies (Knopik & Migawa, 2018; Moghaddass, 2015). This is facilitated by constant technological progress and competition on the market, both of which determine the development of effective and reliable methods of measuring the performance and reliability of devices and equipment. Despite the abundance of such models in the literature, there are no comprehensive descriptions of repair subsystems functioning in complex systems of operations. There are several reasons for this. It is primarily caused by the difficulty of obtaining and processing data for such a study. The data is often available in the form of paper documentation, which is difficult to analyze and requires time-consuming creation of appropriate electronic databases. The second problem, mentioned in publications, is the quality of such information, which makes it difficult to obtain reliable results. The authors draw attention to the lack of appropriate statistical data (necessary variables) (Nowakowski, 2012; Urciuoli, 2011) or their incomplete or fragmentary state (Mlynczak & Nowakowski, 2006). This makes calculation procedures difficult (Szkoda & Kaczor, 2016; Szawłowski, 2008], or even renders obtaining reliable results impossible (Nowakowski, 2006).

Moreover, transport companies, being focused on their core business activities, do not always pay adequate attention to the carried-out repairs, which also means that such analyses are not frequent. This is the reason behind them becoming the basis for the studies presented in this article.

2. RESEARCH METHOD

Randomness of object operation processes, which means that in subsequent moments of time the examined technical object may be in different operating states, allows to apply the theory of stochastic processes. In each of the defined states there is a possibility of modifying it, making corrections and controlling the analyzed process in a way ensuring the highest level of readiness of technical objects treated individually, as well as of the system as a whole. The article analyses the repairs carried out in the company, as a result of their aforementioned worryingly long durations.

Markov and semi-Markov processes are popular in such analyses. In such a case the analyzed object operation process is – from the functional perspective – a process of changes in the distinguished operating states of the object, belonging to a finite set $S = \{S_i;\ i = 1, 2, 3, \ldots, I\}$.. The elements of this set are the values of the process $\{W(t)\colon t\ >\ 0\}$,, i.e. consecutive states $S_i \epsilon S$ remaining in a causal relation [0].

The application of the theory of stochastic semi-Markov processes and, in special cases, the theory of Markov processes requires the fulfilment of specific assumptions.

Their basic property is the memorylessness of the process, which means that the conditional probability distribution of a random variable X_{n+1} depends only on the probability distribution of one of the random variables X_n, and does not depend on the whole past, i.e. on the values that the process assumed in states $i_1, i_2, \ldots, i_{n-1}$ [0].

Conditional probabilities $p_{ij}\colon P(X_{n+1} = j | X_n = i) = p_{ij}$ are called the probabilities of transition from the state i at the moment n to the state j at the moment $n + 1$ (Cui & Wu, 2019).

The form of distributions of the average durations of individual operating states is also of great importance. For Markov processes, they must have an exponential distribution. If these distributions belong to any known parametric family, then it is possible to use the semi-Markov processes theory (Sericola, 1994). If this distribution does not belong to any known parametric family of distributions, perturbed Markov processes theory can be applied (Grabski & Jaźwiński, 2009).

The mathematical model is by definition a simplification of the actual systems of object operation and therefore requires certain assumptions. In the analyzed system it is assumed that the technical object (an automobile) can only be in one of the possible operating states at any given moment of time, the set of these states is discrete and finite, there are no transient states, and the moments of their changes are measurable.

3. PRELIMINARY TESTS

The research sample consisted of data on repairs carried out for the studied transport company. The fleet of vehicles consisted of Scania R420 Euro 5 trucks equipped with manual transmission. As all vehicles came from a single production batch, the sample could be considered as homogeneous. The operation process was analyzed from the moment of transferring the vehicle to the automobile repair shop.

The database was developed using records from repair registers and vehicle service record books. On the basis thereof, a set of operating states was distinguished, presented in the table below (Table 6).

Table 6. Operating states distinguished for the examined technical objects

Operating state	Function
State S1 – physical repair process	The state in which the physical activities associated with restoring the vehicle's operability are performed. It includes not only mechanical repairs, but also body and paint work as well as vehicle maintenance.
State S2 – waiting for a mechanic	The state in which the vehicle, after determining that it is necessary to transfer it to the automobile repair shop, awaits repair. This is due to the schedule of repairs, the occupancy of all repair stations, and sometimes the time of day when the car is transferred (if it is transferred at the end of the work day, repair will start not earlier than on the next day). It is also a state in which repair processes (those started earlier) cannot be continued due to the lack of a mechanic. The mechanic may be involved in activities with another vehicle (which will indicate a shortage of staff) or it is required to call an expert on specific repairs or adjustments. An example of that is a specialist in disassembling damaged and stuck injections and glowplugs, who is able to perform these operations without removing the cylinder head, which will reduce the overall cost of repair.
State S3 – waiting for parts	The state in which repair processes cannot be continued due to the lack of suitable spare parts or subassemblies. This is a result of the organization of the repair subsystem, in which the parts are ordered only if a failure is diagnosed, thus reducing inventory to basic components such as oil filters, air filters, cabin pollen filter, fuel, or brake disks and pads only. There are some subassemblies/parts in the case of which waiting time is usually prolonged. This is the case with powertrain components in particular, e.g. drive shafts, steering system components, such as a steering gearbox. The waiting time may also be extended due to the decision to repair (instead of replacing) a damaged part, e.g. alternator, starter motor, steering gearbox.
State S4 – diagnostics	Diagnosis takes place at the beginning of the repair process, it is then that the scope of repair works is determined. This state covers both the diagnosis of the damaged component or subassembly, as well as preventive diagnostics of the whole vehicle, carried out mainly on the basis of failure testers (diagnostic computers), modern tools and the mechanic’s experience. It is also a state that finalizes the repair process by checking whether all failures were rectified and the vehicle is fully operational.

Source: own elaboration based on the repair register

The distinguished operating states were subjected to detailed statistical analysis. According to the presented assumptions, first of all the so-called memorylessness was checked. It means that the probability of transitioning to any given state at a given moment $t + 1$ depends only on the state at the moment t, and not on the states at previous moments. The homogeneity of an unknown chain can be checked by performing statistical significance tests on randomly calculated differences from a subsample of values of probabilities p_{ij}. If the Markov chain is homogeneous, the distribution of probabilities of transitions between individual states will be constant, expressed in a stochastic matrix of transition probabilities P. The next step was to check whether the time distributions belong to a known parametric family. Such information allows to express the dependencies between the transition probabilities using a formula and facilitates the calculation of basic characteristics of the Markov or semi-Markov process. It also allows to determine which of these models is the most suitable one. Fit to several families of distributions: lognormal, exponential, Weibull and normal was checked using the value of the Akaike information criterion. The distribution for which it reached its lowest value was considered the best. An example of the analysis for the operating state S1 is shown in Table 7.

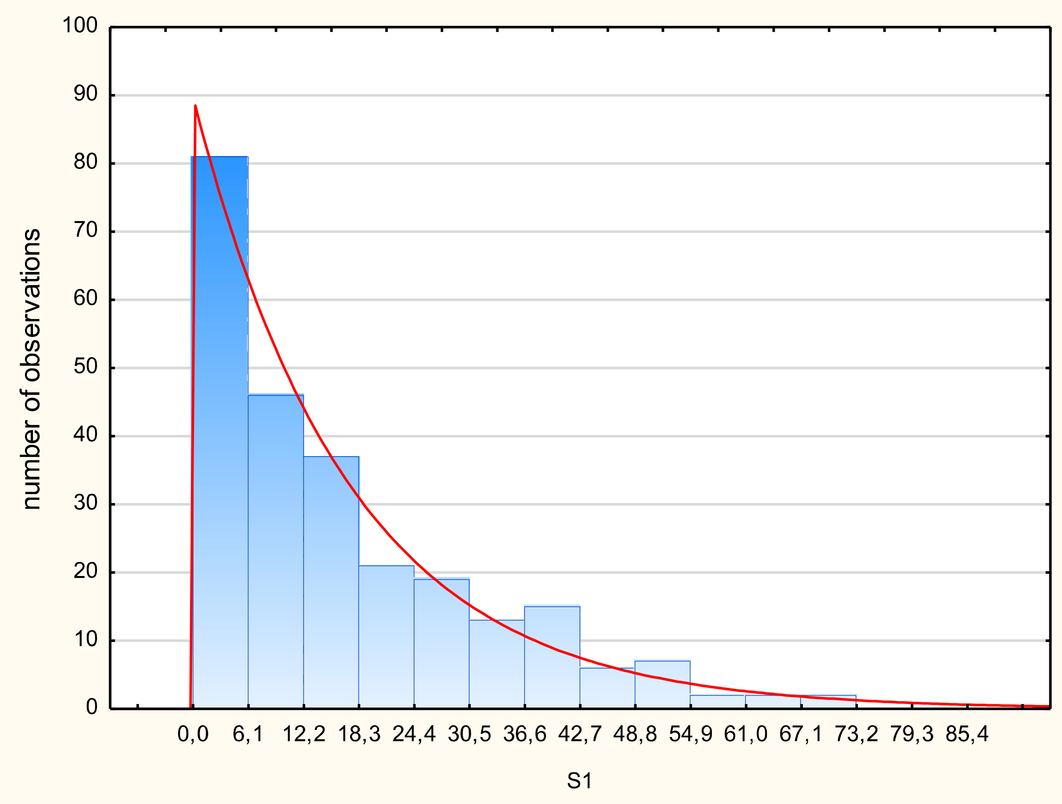
Table 7. Value of Akaike Information Criterion (AIC) and parameters of selected distributions for state S1

Family of distributions	AIC	Parameters				
Log-normal	1964.59	$\mu = 2,31$	$\sigma = 1.20$			
Log-mixtures	1939.90	$\mu_1 = 1.61$	$\sigma_1 = 1.15$	$\mu_2 = 3.13$	$\sigma_2 = 0.58$	$\lambda = 0.54$
Exponential	1929.59	$\lambda = 0.06$				
Weibull	1930.65	$\kappa = 1.05$	$\lambda = 0.06$			
Normal	2095.99	$\mu = 17.11$	$\sigma = 15.62$			

Source: own study

Then the goodness of fit was checked using the Kolmogorov–Smirnov test. The obtained result was not statistically significant, which did not allow to reject the h0 hypothesis of the goodness of fit between empirical and theoretical distribution. The goodness of fit is also confirmed by Figure 24. presenting the empirical distribution of duration times of state S1 and the graph of density of exponential distribution with the parameter $\lambda = 0.06$.

Figure 27. Histogram of time of S1 with the fit of the exponential distribution



Source: own study

Then, the basic measures of descriptive statistics for the analyzed operating states were determined, which are presented in Table 8.

Table 8. Values of basic measures of descriptive statistics of the distinguished operating states

Descriptive statistics						
Variable	Mean [h]	Median [h]	Min. [h]	Max. [h]	Standard deviation [h]	Coefficient of variation [%]
State S1	16.5	12.1	0.2	71.7	15.6	94
State S2	2	1.8	0.5	4	13.9	67
State S3	12	10.7	6	48	20.3	169
State S4	0.9	0.79	0.25	2	9.8	108

Source: own study

The obtained results show a large variation in the duration of individual states, which results from the specificity of the repair process. E.g. for state S1, simple actions related to rectifying an on-board computer error caused by low battery voltage lasted several minutes, hence the minimum value for this state is 0.2 h. On the other hand, long times (maximum value of 71.7 h) usually were related to the body and paint works, which include the time of surface preparation, its painting and waiting for drying or activities related to anti-corrosion maintenance of the chassis, which requires disassembly of many subassemblies, including the exhaust system and fuel supply system.

The collected data is characterized by high variability, resulting from the variety of tasks performed. The mean and the median differ from each other, in all cases the median is lower than the mean value, which is characteristic for exponential distributions.

4. ESTIMATION OF MARKOV MODEL PARAMETERS

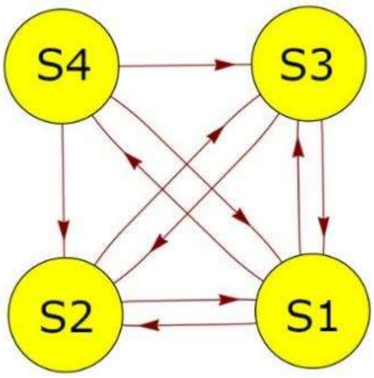
The first stage of constructing the Markov model is the assessment of dependencies between the states by defining permitted and prohibited transitions, marked as $p_{ij.}$. In the examined system, they were determined on the basis of empirical state changes. If in the entire process transitions from state S_i to state S_j took place, they were considered permitted and marked 1, otherwise were prohibited and marked 0. The obtained results are presented in Table 9. and the graph of transitions, is shown in Fig 25.

Table 9. Transition probability matrix of the Markov process

Operating state	S1	S2	S3	S4
S1	0	1	1	1
S2	1	0	1	0
S3	1	1	0	0
S4	1	1	1	0

Source: own study

Figure 28. Graph of relations between states of the studied process



Source: own study

After determining the occurring relations (Fig. 2.), on the basis of the frequency of individual occurrences, the values of elements of the transition probability matrix were calculated. The results are presented in Table 10:

Table 10. Transition probability matrix of the distinguished operating states

Operating state	S1	S2	S3	S4
S1	0	0.12	0.38	0.5
S2	0.89	0	0.11	0
S3	0.78	0.22	0	0
S4	0.34	0.08	0.58	0

Source: own study

The first important characteristic describing Markov chains is their limit properties. They provide information on the process behavior after a long period of time (theoretically when $n \to \infty$). The calculation of limit probabilities requires solving the system of equations (1):

$$\begin{cases} \Pi^T P = \Pi^T \leftrightarrow \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \end{cases} \quad (1)$$

For the process described by 4 operating states, the above formula takes the form (2):

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}^T \cdot \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & p_{32} & 0 & 0 \\ p_{41} & p_{42} & p_{43} & 0 \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix}^T \quad (2)$$

with the normalization condition (3):

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad (3)$$

which is equivalent to the following system of equations:

$$\begin{cases} \pi_2 \cdot p_{12} + \pi_3 \cdot p_{13} + \pi_4 \cdot p_{14} = \pi_1 \\ \pi_1 \cdot p_{21} + \pi_3 \cdot p_{23} = \pi_2 \\ \pi_1 \cdot p_{31} + \pi_2 \cdot p_{32} = \pi_3 \\ \pi_1 \cdot p_{41} + \pi_2 \cdot p_{42} + \pi_3 \cdot p_{43} = \pi_4 \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \end{cases} \quad (4)$$

The solution of the equation [Grabski & Jaźwiński, 2009] are the stationary probabilities π_j presented in Table 11.

Table 11. Limit probabilities of the Markov chain

State	π_j	π_j[%]
S1	0.3968	40
S2	0.1250	13
S3	0.2797	28
S4	0.1984	20

Source: own study

The calculated probabilities express the limit frequencies of the system remaining in particular operating states. The highest value refers to the state in which the vehicles are subject to repair operations. It equals 40% and is, together with state S4, for which the limit value is 20%, a desirable and necessary element of the whole subsystem (60% incidence frequency of all observations). The remaining 40% consist of the frequency of waiting for spare parts and mechanics, the incidence of which should be as low as possible. However, because these are the results concerning the frequency of occurrence of particular states in the set of events, they may not fully reflect the specificity of the process. The parameters calculated for the Markov process for continuous physical time will be more important.

The first step of such analysis is to determine the value of the elements of the transition intensity matrix Λ which is another important characteristic of Markov process. For homogeneous processes, these elements are constant and equal to the opposite of the value of the expected conditional state duration S_i before S_j (5). On the other hand, the intensities $\lambda_{ii} \leq 0$ for $i = j$ are defined as a complement to the sum of the intensities of transitions from state S_i for $i \neq j$ to S_j (Bielecki et al., 2017):

$$\lambda_{ij} = \frac{1}{E(t_{ij})} \tag{5}$$

The first step was to calculate the average conditional duration of particular states. The obtained results, expressed in hours, are presented in Table 12.

Table 12. Expected conditional values of durations of particular operating states [h]

State	S1	S2	S3	S4
S1	0	16.6	16.4	16.5
S2	1.9	0	2.1	0
S3	12.1	11.85	0	0
S4	0.9	0.87	0.99	0

Source: own study

This allowed to calculate, according to the conditional formula (5), the values of transition intensity of the Markov process, presented in Table 13.

Table 13. Transition intensity matrix of the distinguished operating states

State	S1	S2	S3	S4
S1	-0.18	0.06	0.06	0.06
S2	0.53	-1.00	0.48	
S3	0.08	0.08	-0.17	
S4	1.11	1.15	1.01	-3.27

Source: own study

The determined transition intensity matrix allows to determine the limit probabilities p_j in continuous physical time. To this end, it is necessary to solve the system of equations (6):

$$\begin{cases} \Pi^T \Lambda = 0 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{cases} \tag{6}$$

where:

$\Pi^T = [p_j]^T = [\,p_1 ;; \, p_4\,]$ is the transposed (line) vector of limit probabilities p_j of states S_j number $j \in \{1; ; 4\}$,
For the studied process, the equation (6) takes the form:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & 0 \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -\lambda_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{7}$$

Which comes down to solving the system of equations (8):

$$\begin{cases} -\lambda_{11} \cdot p_1 + \lambda_{21} \cdot p_2 + \lambda_{31} \cdot p_3 + \lambda_{41} \cdot p_4 = 0 \\ -\lambda_{22} \cdot p_2 + \lambda_{12} \cdot p_1 + \lambda_{32} \cdot p_3 + \lambda_{42} \cdot p_4 = 0 \\ -\lambda_{33} \cdot p_3 + \lambda_{13} \cdot p_1 + \lambda_{23} \cdot p_2 + \lambda_{43} \cdot p_2 = 0 \\ -\lambda_{44} \cdot p_4 + \lambda_{14} \cdot p_1 = 0 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{cases} \tag{8}$$

The obtained results are presented in table 14.

Table 14. Limit probabilities p_j of the system remaining in states S1-S4 in continuous physical time.

	S1	S2	S3	S4
pj	0.4876	0.0739	0.4296	0.0089
pj %	49	7	43	1

Source: own study

The calculated values P_j are limit probabilities determining that the system will be in a given operating state for a longer period of operation ($t \to \infty$). In this case, the highest probability concerns state S1 – repair, and equals almost 49%. The dominance of this state is a positive result, but value for state S3 is also alarmingly high, at 43%. This means that the limit time spent in the repairs state is almost equal to the duration of the state of waiting for parts. Other results are satisfactorily low, 7% value is generated by delays related to the absence of staff, and diagnostics amounts to only 1%.

The obtained limit probabilities of the Markov process clearly indicate the area requiring correction and the reason for prolonged repair times. It is necessary to reorganize the system of ordering spare parts by, for example, renegotiating contracts with suppliers, increasing the inventory of the most frequently needed components or creating the so-called technical first aid kits including components of the electrical system such as alternators and starter motors, the acquisition of which from the market is often associated with long waiting times, while the replacement process itself is not time-consuming.

The calculated limit probabilities can be presented as a function of time, because they fulfil the Chapman – Smoluchowski – Kolmogorov differential equations. This dependence, in the form of a matrix is represented by the equation (9).

$$\prod^T \Lambda = \frac{d\prod^T}{dt} \tag{9}$$

Which, for the system under examination, takes the form (10),

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \end{bmatrix}^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & 0 \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} & 0 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & -\lambda_{44} \end{bmatrix} = \begin{bmatrix} p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \end{bmatrix}^T \tag{10}$$

solved with the normalization condition (11),

$$\Sigma j \, p_j = 1 \tag{11}$$

therefore, the system of differential equations equivalent to the above relations has the form:

$$\begin{cases} p_1'(t) = -\lambda_{12} \cdot p_2(t) - \lambda_{13} \cdot p_3(t) - \lambda_{14} \cdot p_4(t) + \lambda_{21} \cdot p_2(t) + \lambda_{31} \cdot p_3(\\ p_2'(t) = -\lambda_{21} \cdot p_1(t) - \lambda_{23} \cdot p_3(t) + \lambda_{12} \cdot p_1(t) + \lambda_{32} \cdot p_3(t) + \lambda_{42} \cdot p_4 \\ p_3'(t) = -\lambda_{31} \cdot p_1(t) - \lambda_{32} \cdot p_2(t) + \lambda_{13} \cdot p_1(t) + \lambda_{23} \cdot p_2(t) + \lambda_{43} \cdot p_4 \\ p_4'(t) = -\lambda_{41} \cdot p_1(t) - \lambda_{42} \cdot p_2(t) - \lambda_{43} \cdot p_3(t) + \lambda_{14} \cdot p_4(t) \\ p_1(t) + p_2(t) + p_3(t) + p_4(t) = 1 \end{cases} \tag{12}$$

The solution of this equation, obtained with the assumption that at the initial moment of time $t = 0$ process $X(t)$ is in state S4 – diagnostics, time dependencies of probabilities of observation of states S1-S4 occur, which are described below – equations (13-16):

$$P_1(t) = \frac{-0.27\,e^{1.3\,t} - 0.22\,e^{3.54\,t} - 0.006\,e^{4.4\,t} + 0.49652012919442656 e^{4.6\,t}}{1{,}6 \times 10^{-15} e^{1.3\,t} - 3{,}9 \times 10^{-16} e^{3.5\,t} + 9{,}7 \times 10^{-19} e^{4.4\,t} + 1, e^{4.6\,t}} \tag{13}$$

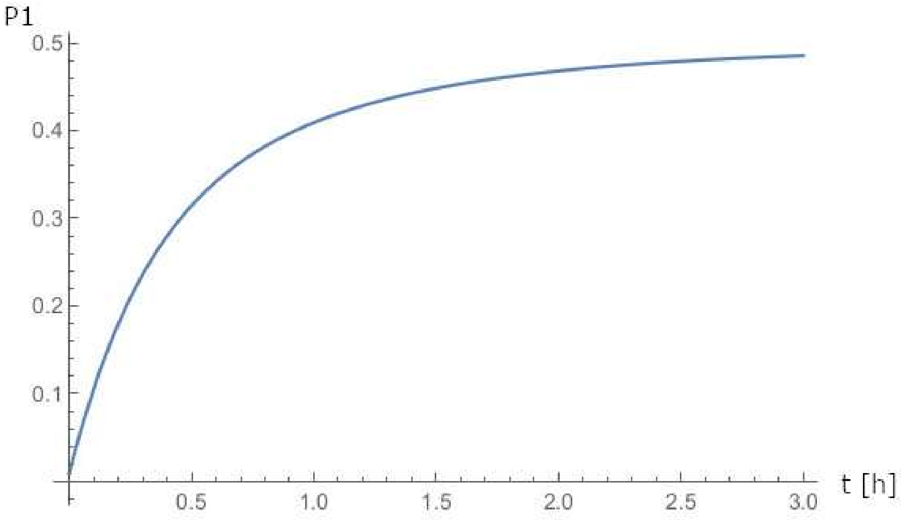
$$P_2(t) = \frac{-0.49\,e^{1.34\,t} + 0.41\,e^{3.54\,t} - 0.000009\,e^{4.37\,t} + 0.07 e^{4.62\,t}}{1{,}55 \times 10^{-15} e^{1.33\,t} - 3{,}9 \times 10^{-16} e^{3.54t} + 9{,}74 \times 10^{-19} e^{4.37t} + 1. e^{4.62t}} \tag{14}$$

$$P_3(t) = \frac{-0.24\,e^{1.33\,t} - 0.19\,e^{3.54\,t} + 0.006\,e^{4.37t} + 0.43 e^{4.62\,t}}{1{,}55 \times 10^{-15} e^{1.34\,t} - 3{,}91 \times 10^{-16} e^{3.54\,t} + 9{,}74 \times 10^{-19} e^{4.37\,t} + 1. e^{4.62\,t}} \tag{15}$$

$$P_4(t) = \frac{0.99\,e^{1.34\,t} - 0.006\,e^{3.54t} - 0.0001\,e^{4.37\,t} + 0.00894630863413386 e^{4.62\,t}}{1.55 \times 10^{-15} e^{1.33\,t} - 3.9 \times 10^{-16} e^{3.54\,t} + 9{,}74 \times 10^{-19} e^{4.37t} + 1. e^{4.62\,t}} \tag{16}$$

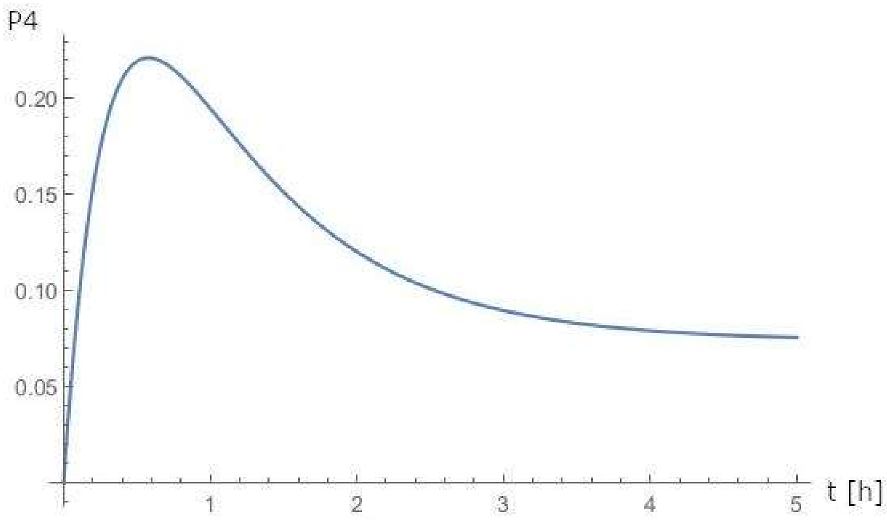
They allow to make a graphical representation of the system's achievement of limit equilibrium. Sample graphs for the state with the highest probability value – S1 and for the state with the lowest value – S4 are shown in Fig. 26 and Fig 27.

Figure 29. Time for reaching the limit equilibrium for state S1



Source: own study

Figure 30. Time for reaching the limit equilibrium for state S4



Source: own study

The results obtained indicate that the system reaches the state of limit equilibrium a few hours after inducement. This means that the processes are stabilized and will not improve without specific and decisive steps. Therefore, it is foremost necessary to thoroughly diagnose the causes of prolonged spare parts waiting times and to take initiatives to improve the process.

5. FINAL THOUGHTS

The company's activity is influenced by many elements, which together define the reliability of the entire system. Its smooth operation requires control over each area, including the one related to the repair of technical objects. It is often a case that only a thorough diagnosis of repair processes exposes their shortcomings. It also allows to indicate directions for improvement. Mathematical methods and tools supporting the evaluation of processes are helpful in this respect.

The article presents the possibility of using one of such tools. Modelling using Markov processes was selected. They are used to describe random processes, which undoubtedly include tasks performed by vehicles. On the basis of the data on repairs provided by the transport company, an assessment of the repair subsystem was made and a long-term forecast was presented for the implementation of the individual states distinguished for the studied system.

The conducted research allowed to indicate the characteristics of the system, especially long stays in the state of waiting for spare parts. In the author's opinion, the problem related to restocking the components necessary to perform repair operations is that it takes too long and it requires detailed analysis and correction. There is a chance that, after appropriate changes, the total time spent by vehicles in the repair subsystem will be reduced.

The aim of the article was also to show that it is possible to apply mathematical modelling for assessment and shaping of the repair subsystem in a vehicle operation system (transport company). Reliable forecasting in this area is important both for the persons responsible for it and for the freight forwarder securing the assigned transport tasks.

In the course of further research directions, primarily it is worth making a detailed analysis of the repair subsystem, which will allow to diagnose the cause of prolonged waiting times for spare parts. In a broader perspective, it is worth considering the possibility of automation and computerization of the presented calculations. This would increase the usefulness of the proposed method by making the form of the presentation of complex mathematical procedures useful for transport system owners/users, e.g. simple mobile applications.

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