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Determining a resilient stope boundary for underground mass mining projects

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Abstract
Uncertainty-based stope boundary optimization is a complex part of underground mine planning, especially in mass mining projects and notably block caving. Besides, grade variation and grade uncertainty are significant sources of error in mining projects. This paper presents a procedure to determine a resilient block-cave stope boundary considering the ore grade uncertainties. The procedure applies the floating stope algorithm, the maximum upside/minimum downside, and the value at risk for design evaluation. The floating stope algorithm is customized for block caving and is used to determine the stope boundary over some simulated grade models. The idea fits into a multi-criteria decision-making problem. Finally, the most resilient stope boundary is selected by considering several criteria and the TOPSIS method. According to the results, the resilient stope boundary covers an area where the mineable reserve is 977 Mt with an average copper grade of 0.51%.

Keywords:
block caving; resilient stope boundary; grade uncertainty

1. Introduction

There is uncertainty in all aspects of a mining project, such as geological, technical, financial, and environmental uncertainties (**Rendu, 2002; Grieco and Dimitrakopoulos, 2007**). Studies in 48 projects in Australia show that in almost 50% of the projects there is a 20% of underestimation or overestimation (**Dimitrakopoulos and Grieco, 2009**). **Vallee (2000)** notes that 73% of mining projects in Canada and the United States have been closed due to reserves estimation errors, causing a loss of about $1.1 billion. Geological uncertainty significantly impacts mine design and planning, highlighting the importance of uncertainty-based mine evaluation and optimization. Therefore, the optimal and resilient mine design should not be sensitive to unexpected circumstances. The effect of uncertainty in underground mining should also be investigated (**Maschio and Schiozer, 2015**). However, due to the variety of underground mining methods and the complexity of underground mining parameters, uncertainty in underground mines has received less attention.

The stope boundary optimization seeks to find a part of a mineral resource that maximizes the mining net economic value. The underground stope boundary optimization algorithms can be divided into rigorous and heuristic algorithms. These algorithms can also be applied in certain and uncertain conditions. Many researchers have worked in mining stope optimization algorithms, most of which have been discussed in terms of specific parameters (**Alford, 1995; Ataee-Pour, 2000; Little et al., 2011; Bai et al., 2013; Sandanayake et al., 2015; Nhleko et al., 2018; Hou et al., 2019; Nikbin et al., 2019; Nikbin et al., 2020**). An underground stope is an area in which rock is extracted using a suitable underground mining method (**Sandanayake et al., 2015**). **Grieco and Dimitrakapoulos (2007)** presented an algorithm for optimizing the stope boundary in conditions of mineral uncertainty. They applied Integer Programming (IP) to determine the optimal number of stopes, size, and location. **Matamoros and Kumral (2018)** presented a stope boundary optimization algorithm that is not sensitive to unexpected accomplishments. This algorithm uses the genetic algorithm to discover the near-optimal solution in the presence of geologic uncertainty. **Faria et al. (2021)** have proposed a two-stage Stochastic Integer Programming (SIP) model to optimize sublevel stope design, which seeks to maximize profit while minimizing the development costs. They used geostatistical simulations to quantify grade variation and uncertainties. **Dirkx et al. (2019)** provide a SIP model for long-term planning in block caving mines concerning grade uncertainty. In order to incorporate grade uncertainty into the strategic mining plan of a cut and fill mining operation, **Huang et al. (2020)** proposed a stochastic mixed-integer programming model. The objective is to maximize the mining project's net present value (NPV) and minimize the production deviation risks. **Grbes et al. (2021)** have evaluated underground mining projects

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focusing on environmental concerns. Recent studies have shown the importance of evaluating and optimizing a mine based on uncertainty.

Among the proposed algorithms, the floating stope is one of the heuristic algorithms proposed for optimizing the stope boundary. This algorithm is a powerful tool for optimizing and analysing the mineable reserve and the geometry of underground stopes. Applications of this algorithm have been reported in uncertain conditions (**Davis and Morrison, 1999; Shenavar et al., 2021**). Decision-making in uncertain conditions requires several decision criteria. Value at Risk (VaR) is a risk assessment method that uses standard statistical techniques and measures the worst expected losses under normal market conditions with a certain level of confidence (**Benninga, 2014**). So, it is one of the most straightforward ways to determine risk. The major drawback of VaR is that it does not consider the tail distribution, so that the result can be misleading (**Rahmanpour and Osanloo, 2016**). Therefore, it must be complemented with other criteria. The other helpful criteria for risk assessment are expected values and the maximum upside/minimum downside. **Dimitrakopoulos et al. (2007)** proposed this approach to design open-pit mines based on geological uncertainty. This method is based on selecting a design that creates the maximum desired risk while the undesirable risk is minimized. This approach can also be used in the design of underground mines.

The present study introduces a comprehensive approach to determine a resilient stope design for the block caving method. The maximum upside/minimum downside and VaR criteria are considered and evaluated for all possible alternatives. In addition to the financial dimension, this approach includes the geological dimension in choosing the stope boundary. Determining a minimum stope dimension is also a significant design parameter for mass mining projects requiring careful study. The minimum stope dimensions are defined according to the materials' physical and geotechnical properties of the mineral deposit (**Topal and Sens, 2010**). Nevertheless, how to calculate the minimum stope dimension in different underground mining methods has received less attention. This paper also provides a method of calculating the minimum stope dimensions for the block caving method.

2. Research method

Determining a resilient block caving stope boundary in the condition of grade uncertainty requires (1) grade simulation and the creation of geological block models, (2) selection of the appropriate optimization algorithm and its customization according to the specifications of the extraction method, and finally (3) selection of the resilient alternative. In general, the steps for determining the resilient stope boundary under grade uncertainty are shown in **Figure 1**.

2.1. Grade simulation

Geostatistical simulation method was introduced in 1970, and since then, it has been widely used in various industries such as mining, environment, oil, and gas (**Dubrule, 2003; Deustch, 2002**). The most crucial feature of geostatistical simulation is producing a set of models and their probability of occurrence. This technique can generate several scenarios for the grade distribution in a deposit so that they all resemble each other. In this paper, the Sequential Gaussian Simulation (SGS) method, suitable for modelling continuous data, has been used for grade simulations. The simulation results are acceptable when they can reproduce the initial histogram and variogram (**Cheuiche et al., 2001**).

2.2. Customizing the optimization algorithm for block caving method

The floating stope algorithm is a heuristic algorithm for determining the optimal Stope Boundary (SB) (**Shenavar et al., 2021**). The inputs of this method include the minimum dimensions of the stope, the cut-off grade, the head grade of the stope, and the dilution calculation method. The objective function can be maximizing the tonnage of rock, metal content, or economic value of the extracted material. The algorithm defines two separate envelopes. The inner envelope is created by sharing the best stopes, and the outer envelope is the union of all possible stopes (**Alford, 1995**). The optimal SB should be located as close as possible to the inner envelope and inside the outer envelope. In locating the SB between the two envelopes, parameters such as the mining method specifications, selective mining requirements, the grade of blocks in the outer envelope, dilution, feed grade, and the remaining resources should be checked. Considering that in this paper, the floating stope algorithm will be used to design the optimal SB for block caving, it is necessary to tune the algorithm parameters based on the specifications of block caving.

The ease of conversion of intact rock into crushed mass is reflected by cavability (**Bullock and Hustrulid, 2001**). Prediction of cavability is one of the critical factors in the success of the block caving method. There are two methods for estimating cavability: experimental and numerical (**Laubscher, 1994; Laubscher and Jakubec, 2001**). Rock mass properties and discontinuities affect the cavability of a rock mass. In addition, environmental, geometric, and operational factors also have a significant impact on rock mass cavability. In experimental methods, rock mass cavability is estimated based on the undercut dimensions. Undercut dimension is defined using hydraulic radius (see **Equation 1**). Rock mass caving begins when the hydraulic radius reaches a critical value (**Laubscher, 2000**). Therefore, the stope's minimum dimensions equal the minimum hydraulic radius required for rock mass caving. The Laubscher diagram defines

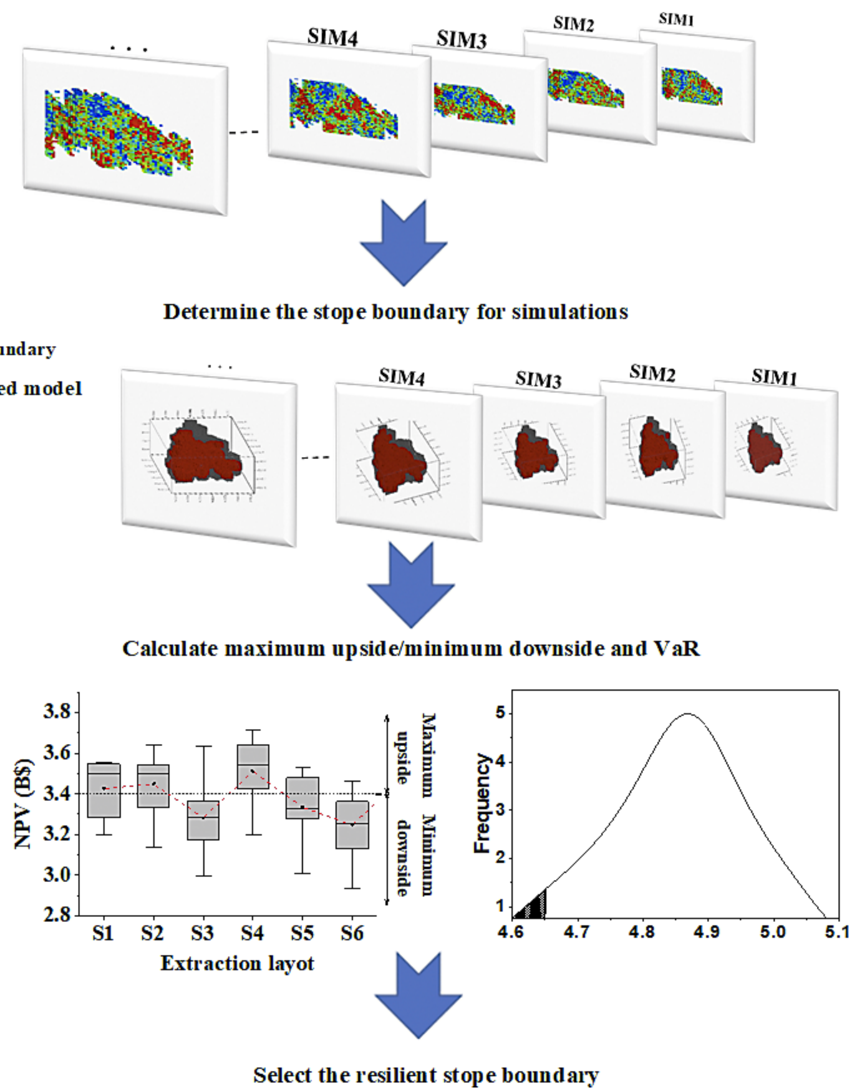


Figure 1: Flowchart to determine the optimal stope boundary in conditions of grade uncertainty

caving or stable situations in terms of hydraulic radius for a range of rock mass conditions (see **Figure 2**).

The *Mine Rock Mass Rating* (MRMR) defines the rock mass condition in this diagram. Then for the given MRMR, the Laubscher caving diagram provides the minimum hydraulic radius for which the caving initiates. In another way, for the given hydraulic radius, the Laubscher caving diagram provides the critical MRMR for which the caving initiates. In that regard, **Equation 2** provides the critical MRMR.

$$HR = \frac{A}{P} \tag{1}$$

$$MRMR_C = -0.0142 \times HR^2 + 2.2211 \times HR - 0.3446 \tag{2}$$

Where:

- HR – hydraulic radius (m),
- A – stope area (m),
- P – stope perimeter (m),
- $MRM\ R_C$ – the critical MRMR for which the caving process starts.

Equation 2 is derived from the Laubscher caving diagram. In this equation, for a given HR, one could calculate the critical MRMR for which the caving process starts (i.e. $MRM\ R_C$). If the MRMR of the region is greater than $MRM\ R_C$, then the region is stable, and if the MRMR is lower than $MRM\ R_C$, then the region will start to cave, assuming the given hydraulic radius (i.e. **Equation 3**).

$$\begin{cases} if\ MRMR \geq MRM\ R_C \Rightarrow the\ stope\ is\ stable \\ f\ MRMR < MRM\ R_C \Rightarrow the\ stope\ will\ start\ to\ cave \end{cases} \tag{3}$$

After determining the minimum length and width of the stope, the minimum height should be determined. The stope height is inversely related to the development cost; the higher the stope height, the lower the development costs. However, for higher stope heights, managing the draw control and dilution is difficult (**Tobie and Julin, 1998**). Moreover, the lifespan of draw points imposes these limitations. Therefore, the minimum height of the stope can be considered equal to the height at

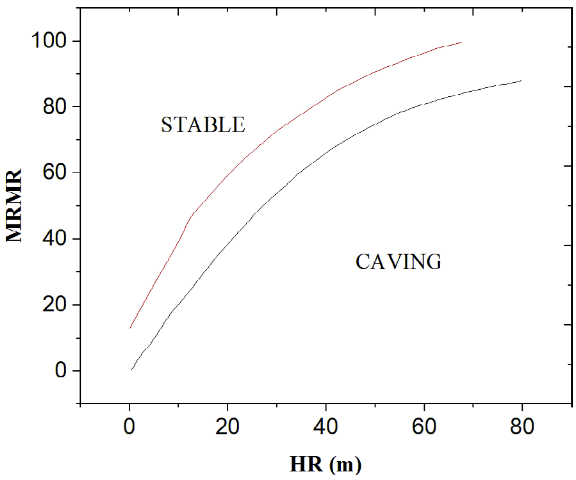


Figure 2: Experimental Laubscher caving diagram (modified from **Laubscher, 2000**)

which the extracted mineral can pay the development cost and the minimum expected profit. Therefore, development costs must be estimated. There are several methods for mine cost estimation (**O'hara and Suboleski, 1992; Hodel et al.,1987; Longerstaey and Spencer, 1996; Doneva et al., 2015**). The minimum stope height is the height for which the minimum expected profit is achieved. Hence, a trial-and-error approach is applied to determine the minimum stope height.

The head grade is another parameter that is required to determine the SB. A mining cut-off grade is a grade that covers the extraction, processing, smelting, and refining costs. The head grade is determined according to the plant feed grade and the cut-off grade.

Another parameter required by the algorithm is the stope floating ranges. The output of the floating stope algorithm is a regular block model in which the dimensions of the blocks are determined according to the minimum stope size. This way, the block dimensions are obtained by dividing the minimum stope dimension by the floating ranges. In this paper, the centre of blocks represents draw-points, and the distance between the draw points is used as if they are floating ranges.

2.3. *Resilient stope boundary determination*

Determining a Resilient Stope Boundary (RSB) is a multi-criteria decision-making problem. In this section, the selection procedure and the criteria used are explained.

2.3.1. TOPSIS selection procedure

The mining sector aims to create maximum value for shareholders and supply raw materials for downstream industries. However, the uncertainty in the mining industry puts some limits on achieving these aims. In that regard, the most resilient alternative should be selected from the available alternatives according to the calculated criteria. The selection of the RSB is a multi-criteria decision-making problem (**Saaty, 1990; Hwang and Yoon, 1981; Toloie-Eshlaghy and Homayonfar, 2011**). The RSB selection depends on the decision-maker and his behaviour. Recently, several mathematical methods have been developed to select the best alternatives. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision-making method used to select the best alternative based on several criteria. In this method, m alternatives are evaluated by n criterion. TOPSIS defines the positive ideal solution and the negative ideal solution. A positive ideal solution increases the profit criterion and decreases the cost criterion. The optimal alternative is the one with the shortest distance from the positive ideal solution and, simultaneously, the farthest distance from the negative ideal solution. The steps of the TOPSIS method are as follows (**Pavic and Novoselac, 2013; Papathanasiou and Ploskas, 2018**):

Step 1: Normalize the decision matrix (**Equation 4**):

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{4}$$

Where:

x_{ij} – the value of the i^th^ SB with respect to the j^th^ criteria,

n_{ij} – the normalized value of x_{ij}.

Step 2: Create the weighted normalized decision matrix (**Equation 5** and **6**):

$$\sum_{j=1}^{n} w_j = 1,\ j = 1,2,\ldots,n \tag{5}$$

$$v_{ij} = w_j \times n_{ij},\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{6}$$

Where:

w_j – the weight of the j^th^ criteria,

v_{ij} – the weighted normalized value.

Step 3: Determine the positive and negative Ideal Solutions (IS) (**Equation 7** and **8**):

$$\alpha_j^+ = \left\{\left(\max_j v_{ij} \mid i\varepsilon I'\right),\left(\min_j v_{ij} \mid i\varepsilon I''\right)\right\} i = 1,2,\ldots,m,\quad j = 1,2,\ldots,n \tag{7}$$

$$\alpha_j^- = \left\{\left(\min_j v_{ij} \mid i\varepsilon I'\right),\left(\max_j v_{ij} \mid i\varepsilon I''\right)\right\} i = 1,2,\ldots,m,\quad j = 1,2,\ldots,n \tag{8}$$

Where:

α^+ – the positive ideal solution,

α^- – the negative ideal solution,

I' – represents the benefit criteria,

I'' – represents cost criteria.

Step 4: Calculate the distance from the positive and negative IS (**Equation 9** and **10**):

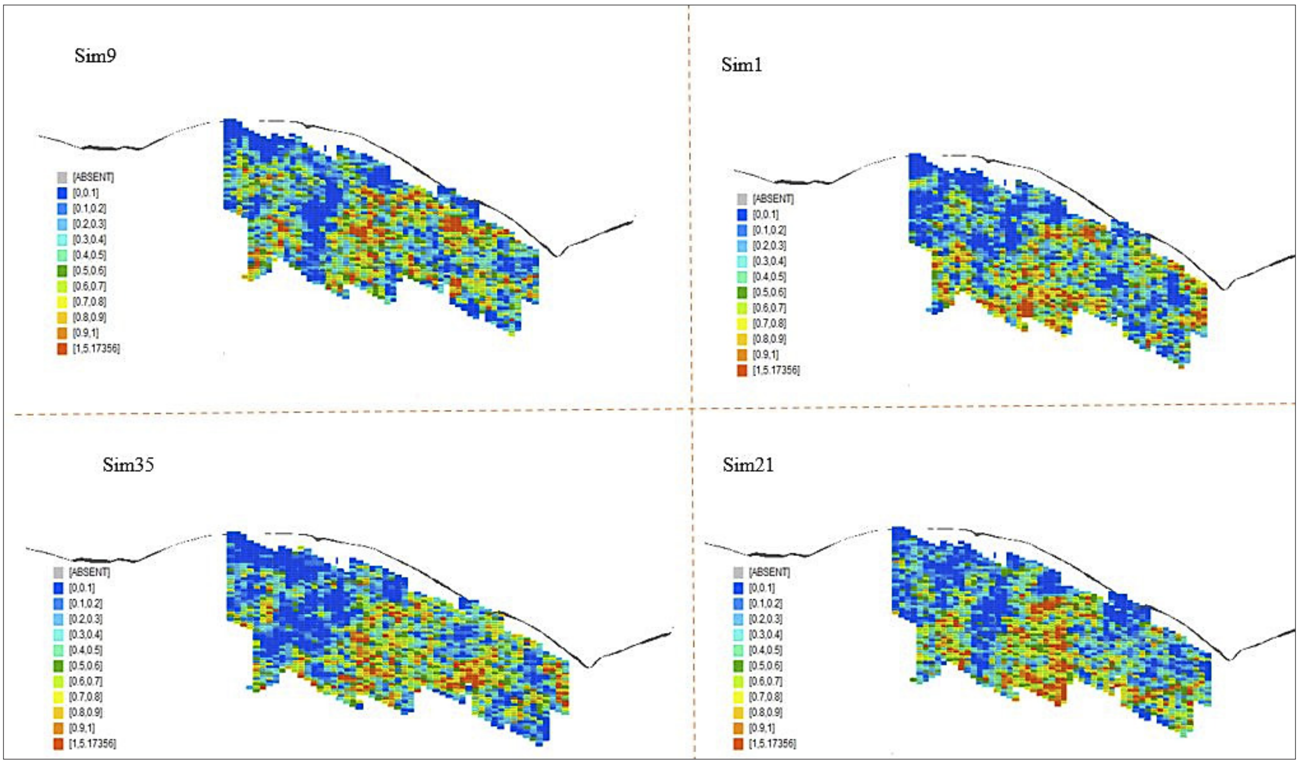


Figure 3: Four realizations from 40 simulated grade block models

$$d_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - \alpha_j^+\right)^2},\ i = 1,2,\dots,m,\ j = 1,2,\dots,n \tag{9}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - \alpha_j^-\right)^2},\ i = 1,2,\dots,m,\ j = 1,2,\dots,n \tag{10}$$

Where:

d_i^+ *and* d_i^- – the distances from positive and negative IS, respectively.

Step 5: Calculate the relative distance from the IS (**Equation 11**):

$$D_i^+ = \frac{d_i^-}{d_i^+ + d_i^-},\ i = 1,2,\dots,m \tag{11}$$

Step 6: Sort the alternatives based on the relative distance D_i^+ from largest to the smallest values and select the first rank.

2.3.2. Selection criteria

This study proposes the integration of Value at risk (VaR) and Maximum Upside/Minimum Downside (MU/MD) to determine an RSB according to the aims of the mining sector. These criteria are defined separately for mineable reserve and net present value. VaR is a widely used risk management tool that answers the question: How much can we lose over a pre-set horizon with for a known probability (**Sandanayake et al., 2015**)? (**Equation 12**).

$$VaR_\alpha(V) = \inf\inf\left\{V' \mid pr(V > V') > \alpha\right\} \tag{12}$$

Where:

$VaR_\alpha(V)$ – the value at risk of variable V at a confidence level of α, and $\alpha \in (0,1)$,

pr – the probability,

$\inf\{B\}$ – the greatest number that is less than or equal to the elements of set B.

As stated, **Dimitrakopoulos et al. (2007)** presented the MU/MD approach for optimizing open pit mines. The respective values of MU/MD are defined in **Equations 13 and 14**.

$$MU_i = \sum_{j=1}^{10}(V_{ij} - \bar{c})p_j \ \ if\ V_{ij} \ge \bar{c} \tag{13}$$

$$MD_i = \sum_{j=1}^{10}(\bar{c} - V_{ij})p_j \ \ if\ V_{ij} < \bar{c} \tag{14}$$

Where:

V_{ij} – the value of an indicator for the i^th^ simulation and the j^th^ SB,

$\bar{c}$ – a threshold defined for the intended indicator,

p_j – the probability of occurrence for simulation j,

MU_i – the maximum upside value of the given SB_i

MD_i – the minimum downside value of the given SB_i.

3. Results and Discussions

In this section, the results are presented and discussed in three subsections. At first, grade simulations are generated in a case study. After that, customizing the optimization algorithm is explained, and the necessary parameters are presented. Finally, the alternatives are assessed, and a resilient stope boundary is determined.

3.1. Grade simulations

Songun copper deposit is located in the East Azerbaijan Province of Iran. The distance of this deposit from Tabriz is 147 km. The copper deposit is located in a mountainous area west of the Songun River. The deposit is currently mined by open-pit mining. However, considering the depth of the mineral resource, it is inevitable to transition to underground mining methods such as the block caving mining method. This case is selected for further studies and evaluation of the proposed procedure.

Before simulating the mineral resource by SGS, it is necessary to normalize the data. The semivariogram of normalized data was used. A spherical mathematical model is fitted in three main, sub, and vertical directions, as given in **Equations 15-17**.

$$\gamma(h)=\begin{cases}0.84\left(\frac{3h}{2\times 519}-\frac{h^3}{2\times 519^3}\right)+0.16 & h\le 519\\ 1 & else\end{cases} \quad (15)$$

$$\gamma(h)=\begin{cases}0.84\left(\frac{3h}{2\times 430}-\frac{h^3}{2\times 430^3}\right)+0.16 & h\le 430\\ 1 & else\end{cases} \quad (16)$$

$$\gamma(h)=\begin{cases}0.84\left(\frac{3h}{2\times 378}-\frac{h^3}{2\times 378^3}\right)+0.16 & h\le 378\\ 1 & else\end{cases} \quad (17)$$

Where:

h – distance,

γ(h) – semivariogram.

Based on the data, 40 simulations are generated using the SGS algorithm. **Figure 3** shows some cross-sections of simulated block models. Histograms and semivariograms of these simulations are also calculated (see **Figure 4**). A comparison of the simulated and the fitted model shows that the simulations have been able to reproduce the geostatistical parameters of the region and are valid. In this regard, one could use these simulations for further analysis.

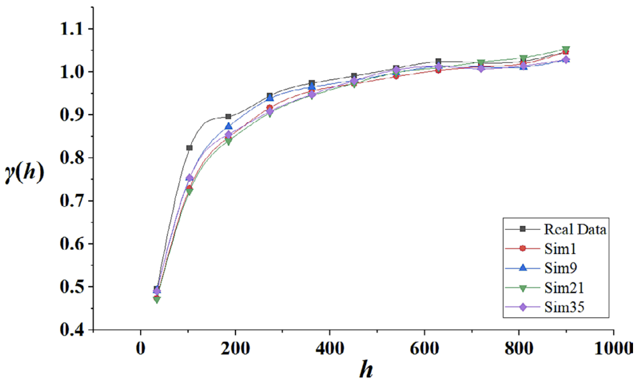


Figure 4: Semivariograms produced by the simulations and the real data

3.2. Customizing optimization algorithm

According to the previous geotechnical works in Songun, the average MRMR is 43. Considering the MRMR of 43 and the Laubscher model (**Figure 2** and **Equation 2**), the minimum hydraulic radius to trigger the caving process is 23 meters. In order to achieve a space with a hydraulic radius of 23 meters, a space with an area of 9.8 thousand square meters must be created. Therefore, the minimum length and width of the stope are considered to be 140×70 meters, according to **Figure 2** and **Equation 1**. Also, the largest dimension of the stope is considered perpendicular to the direction of maximum continuity in the deposit.

The minimum height of the stope is the height that can meet the costs associated with the stope, which is determined by a trial-and-error approach. In this approach, the O'Hara method is used for cost estimation. According to the calculations, the maximum number of development levels for the Songun Copper Mine is 9. Since the height of the mineral resource is 900 meters (i.e. the highest level of mineral 2342m and the lowest level of mineral 1420m), the minimum height of the stope is set to 100 meters. The mining cut-off grade is 0.2%. The minimum head grade is 0.4%, according to the plant requirements. Also, the horizontal floating range is obtained by dividing the stope width and length by the distance between the draw points. Here, the draw points are designed at an interval of 17.5 meters. The draw points are located in the centre of each block to facilitate caving management and grade control. The techno-economic data required to determine the minimum stope height is given in **Table 1**.

After determining the parameters, it is possible to determine the optimal SB for the initially simulated block models. The simulations show that simulations 1, 2, 6, 9, 17, 19, 21, 31, 35, and 37 have the most significant difference in the average grade. Therefore, these simula-

Table 1: Information required to determine the minimum block caving stope height

Parameter name	Value
Extraction and development costs	1.9 \$/ton
Processing cost	6.3 \$/ton
Capital cost	1.4 \$/ton
Overhead cost	0.45 \$/ton
Total recovery	85 %
Cost of smelting and refining	430 \$/ton cu
Copper prices	6000 \$/ton cu
Minimum expected profit	400 M\$
Block dimensions in resource model	17.5×17.5×12.5 m
Cut-off grade	0.2%
Head grade	0.4%
Minimum stope dimensions in the direction of X, Y, Z	70, 140, 100 m

Table 2: VaR (10%) of NPV and metal content for the initially selected alternatives

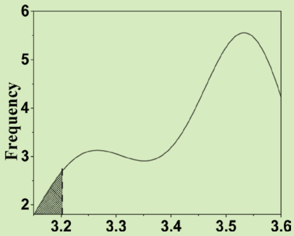
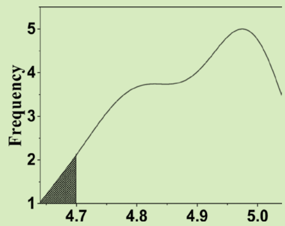
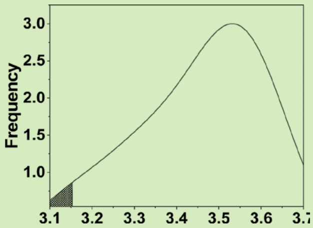
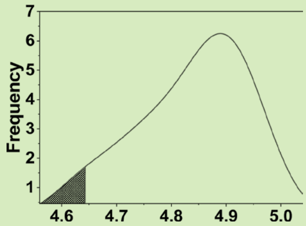
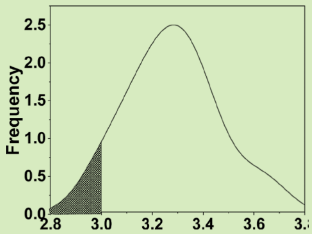
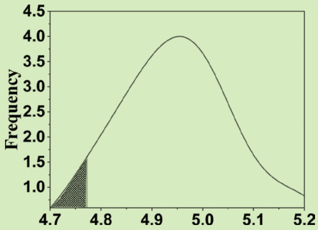
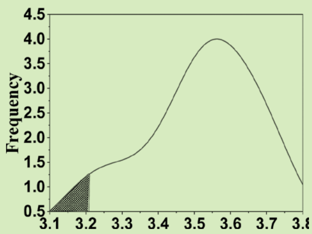
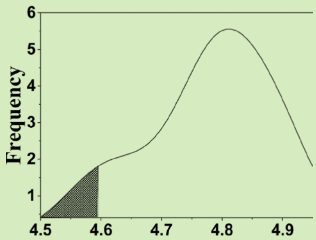
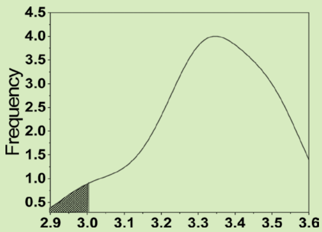
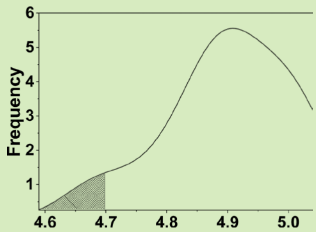
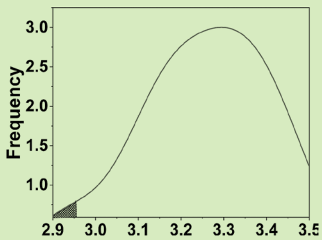
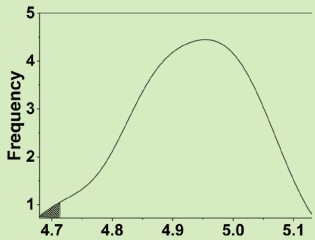
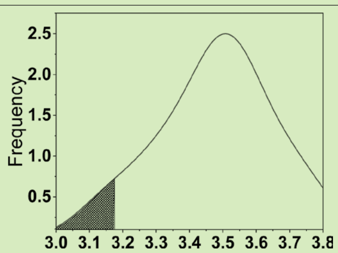
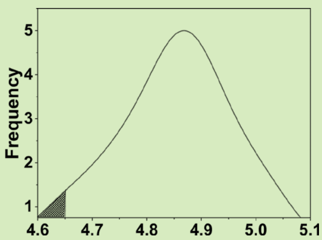
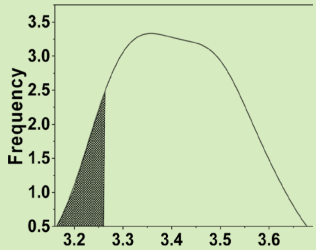
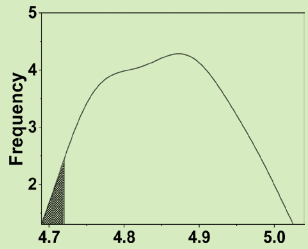
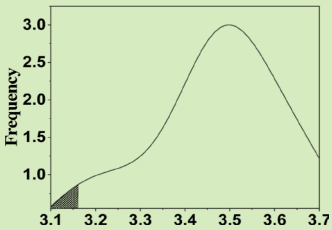
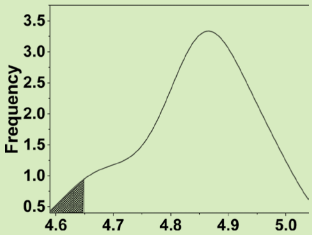
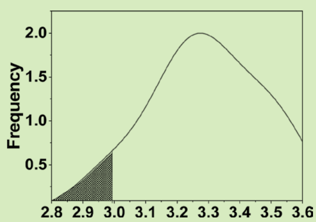
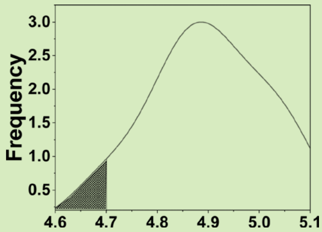
SB	Distribution		Var (10%)	
	NPV (B$)	Metal content (Mt)	NPV (B$)	Metal content (Mt)
Sim1			3.21	4.69
Sim2			3.15	4.64
Sim6			3	4.77
Sim9			3.21	4.59
Sim17			3.02	4.69
Sim19			2.95	4.72
Sim21			3.17	4.65

Table 2: Continued

Sim31			3.26	4.74
Sim35			3.16	4.65
Sim37			2.99	4.7

tions are selected for further analysis. For each of these simulations, the optimal SB is determined using the customized floating stope algorithm and the parameters given in **Table 1**. The process will eventually produce 10 SBs treated as the primary alternatives.

3.3. RSB determination

This section explains the procedure for determining a resilient stope boundary. At first, the initial assessment of alternatives is discussed. After that, the selection process and validation of results are presented.

3.3.1. Initial assessment of alternatives

Several indicators are defined to assess the alternatives. These indicators are VaR(10%) and MU/MD assessments for NPV and metal content. The distribution of NPV and metal content and the corresponding VaR(10%) and MU/MD values for the initially selected alternatives are shown in **Table 2**. For the critical NPV criteria, the SB determined for Sim31 is low-risk, meaning that the probability of achieving an NPV of $3.26 billion is 90% for this case, which is the maximum NPV that can be achieved over others. However, from a metal content point of view, sim6 is low-risk because the probability of achieving a metal content of 4.7 Mt is 90%, which is the maximum metal content that can be achieved over others.

Figure 5 shows the performance of 10 SBs concerning all simulated models. The average NPV of all SBs is used as a threshold in calculating the MU/MD. The purpose of considering this threshold is to obtain a boundary in which the probability of NPV is greater than the average. The average NPV is $3.4 billion.

MU/MD values for different boundaries are calculated according to **Equations 5** and **6**, and the results are given in **Table 3. Figure 5** shows that the NPV varies from 2.93 to 3.75 billion dollars for all SBs. The range of NPVs in Sim1 and Sim31 is less than in others, where the range is almost constant. Among these 10 SBs, Sim9 has the most MU and minor MD risks.

The distribution diagrams of the metal content for different SBs are shown in **Figure 6**. Comparing this dia-

Table 3: Mineable reserve, and MU/MD values for NPV and metal content in 10 SBs

SB	Tonnage (Mt)	MU/MD	
		NPV (B$)	Metal Content (Mt)
Sim1	995	0.09/-0.02	0.05/-0.04
Sim2	979	0.1/-0.04	0.02/-0.06
Sim6	1021	0.03/-0.14	0.10/-0.01
Sim9	958	0.15/-0.03	0.00/-0.10
Sim17	1006	0.03/-0.09	0.05/0-0.03
Sim19	1019	0.01/-0.15	0.07/-0.02
Sim21	977	0.13/-0.03	0.04/-0.05
Sim31	982	0.06/-0.04	0.03/-0.04
Sim35	974	0.11/-0.04	0.02/-0.05
Sim37	1010	0.03/-0.013	0.06/-0.03

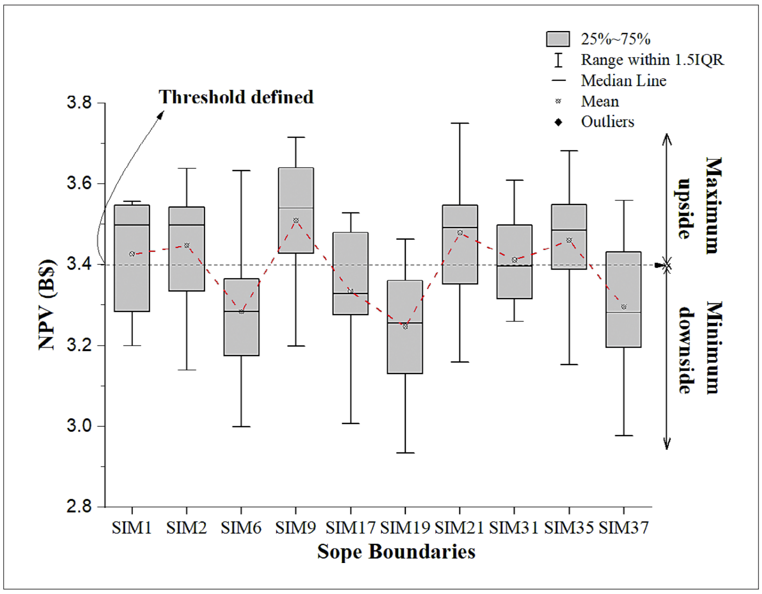


Figure 5: NPV distribution in 10 initial SBs

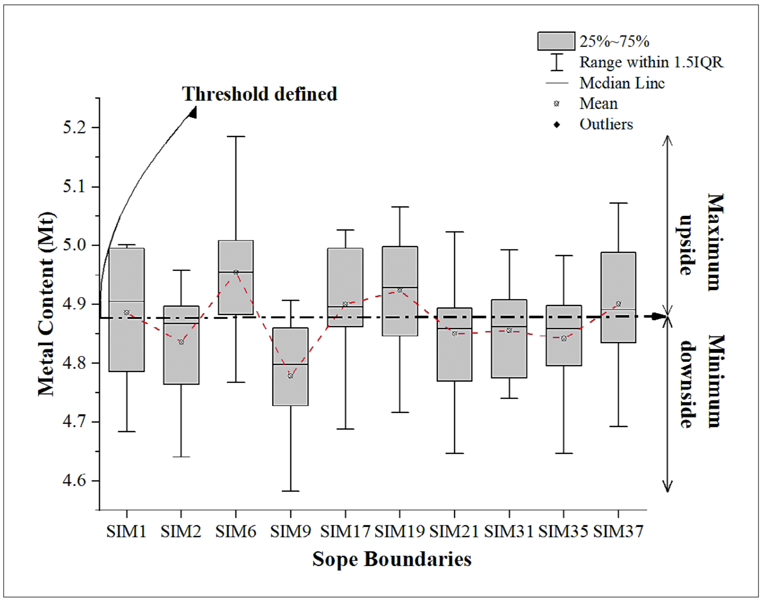


Figure 6: Distribution of metal content in different stope boundaries

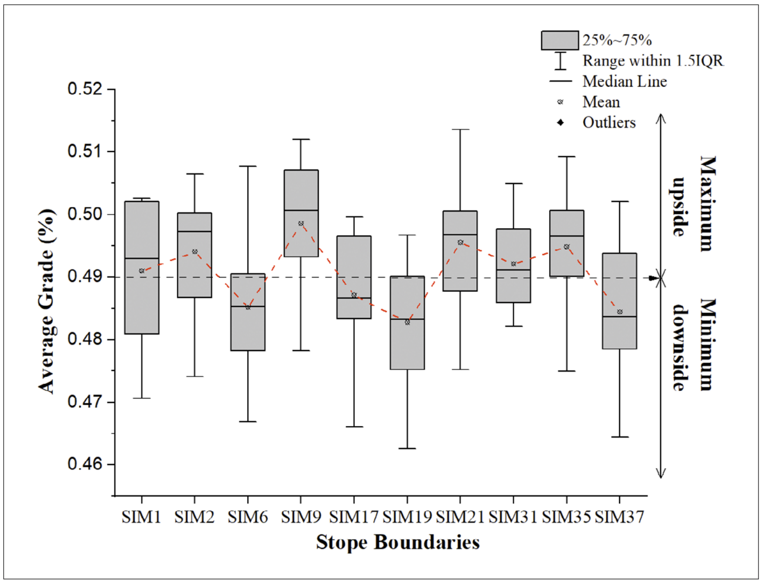


Figure 7: Average grade distribution in different stope boundaries

Table 4: Decision matrix

SB	MU		MD		VaR(10%)	
	NPV (B$)	Metal content (Mt)	NPV (B$)	Metal content (Mt)	NPV (B$)	Metal content (Mt)
Sim1	0.09	0.05	-0.02	-0.04	3.21	4.69
Sim2	0.10	0.02	-0.04	-0.06	3.15	4.64
Sim6	0.03	0.10	-0.14	-0.01	3.00	4.77
Sim9	0.15	0.00	-0.03	-0.10	3.21	4.59
Sim17	0.03	0.05	-0.09	-0.03	3.02	4.69
Sim19	0.01	0.07	-0.15	-0.02	2.95	4.72
Sim21	0.13	0.04	-0.03	-0.05	3.17	4.65
Sim31	0.06	0.03	-0.04	-0.04	3.26	4.74
Sim35	0.11	0.02	-0.04	-0.05	3.16	4.65
Sim37	0.03	0.06	-0.01	-0.03	2.99	4.7

Table 5: Final ranking of SBs

SB	d_i^+	d_i^-	D_i^+	Ranking
Sim1	0.097	0.135	0.583	2
Sim2	0.137	0.11	0.445	8
Sim6	0.129	0.171	0.57	3
Sim9	0.17	0.141	0.454	6
Sim17	0.14	0.102	0.422	9
Sim19	0.154	0.125	0.449	7
Sim21	0.1	0.145	0.593	1
Sim31	0.137	0.100	0.420	10
Sim35	0.132	0.119	0.474	5
Sim37	0.126	0.132	0.511	4

Table 6: Tonnage-grade of the inner and outer envelopes

Parameter name	Value	Parameter name	Value
Mineable reserve in IE*	697 Mt	Average grade in IE	0.63%
Mineable reserve in OE**	280 Mt	Average grade in OE	0.27%
Mineable reserve in total	977 Mt	Average grade in total	0.52%

* IE: Inner Envelope
** OE: Outer Envelope

gram with the diagrams given in **Figure 5** shows that the distribution of these two diagrams is different. A clear example is the SB for Sim9, which has the best performance in terms of NPV but has the worst performance in terms of metal content (see **Table 3**). The reason is the non-selectivity of the block caving method. Since the block caving method is not a selective mining method, this causes the extraction tonnage to remain constant in each SBs while the ore grades are different. The mineable reserves in different SBs are given in **Table 3**. As can be seen, boundary of Sim9 has the lowest extraction tonnage, but according to the average grade (see **Figure 7**), the MU of the average grade is high, and the MD is low. Therefore, the MU of NPV has increased. On the other hand, the low extraction tonnage in Sim9 compared to others has increased the MU of metal content.

3.3.2. RSB evaluation and selection

Now the decision-maker must select the most resilient stope boundary from the available alternatives. For this purpose, the TOPSIS method has been used. The initial decision matrix used for RSB selection is provided in **Table 4**. This table summarizes 10 alternatives, 6 criteria, and their respective values.

In this case, VaR and the MD are two negative criteria, and the MU is the positive criterion. Therefore, assuming an equal weight for the positive and negative criteria, VaR, MU, and MD weights are 25%, 50%, and 25%, respectively. The weighted normalized matrix is evaluated using **Equations 7** and **9**. Then, applying **Equations 10-13**, positive and negative ideal solutions and distances from these solutions are calculated. Finally, the relative distances to the ideal solutions are deter-

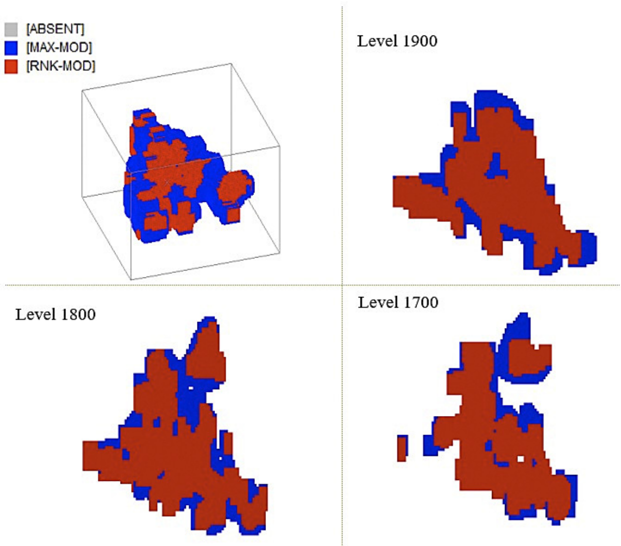


Figure 8: A view of the inner (red) and the outer envelope (blue) and some horizontal sections

mined by **Equation 14** (see **Table 5**). According to the results, Sim21 has been selected as the RSB. The alternative rankings are also provided in **Table 5**. Also, information about the envelopes is given in **Table 6**. According to this table, the grade of the inner envelope is higher than the outer envelope, and combining the envelopes does not cause much change in the average grade. Also, since the grade of the outer envelope is higher than the design cut-off grade and due to the nature of the block caving method, both envelopes can be selected as the stope boundary. A view of the inner and outer envelopes is shown in **Figure 8**.

Table 7: MU/MD and VaR for NPV and metal content in Base case

MU		MD		VaR (10%)	
NPV (B$)	Metal content (Mt)	NPV (B$)	Metal content (Mt)	NPV (B$)	Metal content (Mt)
0.17	0	-0.01	-0.18	3.28	4.51

3.3.3. Validation of the selected RSB

In order to validate the results, another stope boundary is determined conventionally based on the kriging estimates. For the SB obtained from the kriging estimate (Base case), MU/MD and VaR are determined. The results of these calculations are shown in **Table 7** and **Figure 9**. As shown in **Figure 9**, the Base case has performed well for the NPV key indicator in terms of maximum upside (see **Figure 9-B**), while it has performed cheap for the metal content key indicator, with a value of zero (see **Figure 9-A**). In order to compare the Base case with the 10 simulated SBs, the TOPSIS method is used and d_i^+, d_i^- and D_i^+ are shown in **Figure 10**. As shown in this Figure, Sim21 is still the most resilient boundary, while the Base case has the worst performance among the available boundaries. So, the selected Sim21 is valid. Moreover, the Sim21 boundary is compared with the base case stope boundary obtained in deterministic conditions (see **Table 8**). According to **Table 8,** the mineable reserve in Sim21 is higher, and the NPV is approxi-

Table 8: Comparing Sim21 and the deterministic stope boundary

Stope ID	Conditions	Mineable reserve (Mt)	Average Grade (%)	NPV (B$)	Mine life (years)
Base case	Deterministic	931	0.51	3.76	26.5
Sim21	Uncertainty based	977	0.51	3.75	28

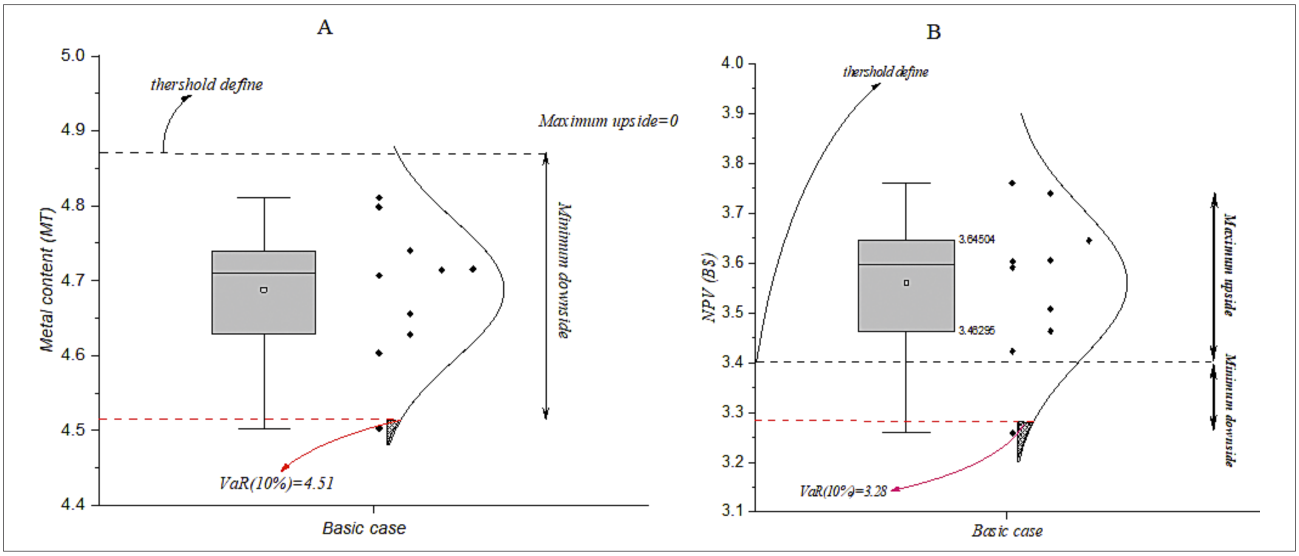


Figure 9: Distribution MU/MD and VaR (10%) for (A) Metal content and (B) NPV

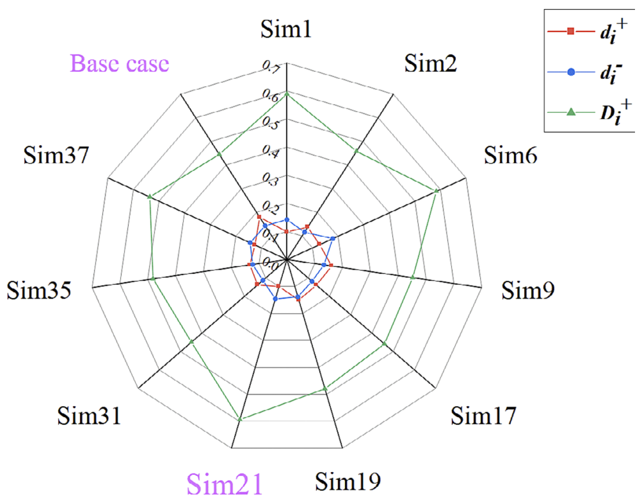


Figure 10: Final result for all realizations

mately equal to the boundary obtained in deterministic conditions.

4. Conclusions

Mining operations will focus on deepening as mineral resources in near-surface deposits are depleted. Among the underground mining methods, the block caving method is competitive in production and costs with surface mines. The first step in planning underground mines is to determine a stope boundary. A resilient stope boundary can better predict the actual results. This paper presents the application of the floating stope algorithm to determine the optimal block caving stope boundary in grade uncertainty conditions. In that regard, 40 simulations were generated, and 10 simulations were selected. Then, the optimal stope boundaries were determined using a customized floating stope algorithm. The algorithm is customized for block caving initially. This paper shows that the minimum dimensions of a block caving stope depend on the minimum hydraulic radius required for caving, extraction costs, and projected profits. The NPV and content metal indicators assisted decision-making in uncertain conditions. Finally, Sim21 was selected as the resilient stope boundary. Based on the results, the mineable reserve is about 977 Mt with an average copper grade of 0.513%.

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SAŽETAK

Određivanje elastične granice iskopa kod velikih podzemnih rudarskih projekata

Optimizacija granica iskopa metodom temeljenom na nesigurnosti složen je dio projektiranja podzemnih rudnika, posebno u projektima masovnoga rudarenja te u metodi blokovskoga iskopa. Osim toga, varijacija u stupnju i nesigurnost sadržaja rude česti su izvori pogrešaka u rudarskim projektima. Ovaj rad predstavlja postupak za određivanje elastične granice blokovskoga iskopa uzimajući u obzir nesigurnosti sadržaja rude. Postupak primjenjuje algoritam plutajućih granica iskopa, maksimalno dobru / minimalno lošu stranu i rizičnu vrijednost za ocjenu dizajna. Algoritam plutajućih granica iskopa prilagođen je za blokovsku metodu iskopa i koristi se za određivanje granice iskopa preko određenih simuliranih stupnjevitih modela. Ideja se uklapa u višekriterijski problem odlučivanja. Konačno, odabire se najelastičnija granica iskopa uzimajući u obzir nekoliko kriterija i TOPSIS metodu. Prema rezultatima elastična granica iskopa pokriva područje gdje eksploatacijske rezerve iznose 977 Mt s prosječnim sadržajem bakra od 0,51 %.

Ključne riječi:
metoda blokovskoga iskopa, elastična granica iskopa, nesigurnost sadržaja rude

Author's contribution

Mohammad Shami-Qalandari (MSc student) provided the reserve simulations and mineable reserve calculations, analysis, and presentation of the results. **Mehdi Rahmanpour** (Assistant Professor) provided the structure of the work, criteria definition and calculations, and interpretations of the results. **Mirabedi, S. M. Mahdi** (Assistant Professor) provided data preparation and the selection method used.