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Modeling of watershed basins using weighted graphs and presenting an algorithm to select suitable sample locations for reducing sampling and analysis costs

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Abstract

Watershed sampling is one of the most important prospecting methods applied for mineral exploration. The objective of this study is mathematical modeling of watersheds for proper sampling and optimization of sample numbers for reduction of sampling and analysis costs. For this purpose, the mathematical modeling of the catchment basins of the studied area was carried out using DEM in GIS by constructing a directed graph with weighting of the upper catchment basins. Then the matrix model of the graph was prepared and assigned weight based on the relevant indexes using codes in the MATLAB and outlet basins and their related weights were determined. In the next step, critical paths were determined in the simulated graph and in each critical path, the key basins were determined for sampling. The proposed algorithm was implemented on a watershed network as a case study and the results showed that this algorithm is able to reduce the costs of sampling and analysis by one quarter.

Keywords: watershed; weighted graph; directed graph; sampling; analysis

1. Introduction

The study of watersheds and stream sediments in order to explore mineral resources, could be carried out by modeling and sampling of the catchments. In all these cases, the goal can be achieved using sampling from the watersheds and analysis of the samples to conclude the results. However, due to the vast network of watersheds, it is not possible to sample and analyze the samples from all of the catchments. The traditional methods are time consuming and require high cost which is not cost effective. Over the years, many studies have been conducted in this field that the main purpose of all these studies is to find the optimal points for sampling in order to reduce the number of samples and consequently reduce the cost and time to reach the desired points. Therefore, the issue of designing and modeling the network of watershed and catchments to find the optimal sampling points has been raised for many years.

In this regard, for the first time, Sharp was able to find the optimal points for sampling by specifying the topological center of the watershed network **(Sharp, 1971)**. The theory of this method was first proposed by some researchers and its implementation began in practice in the seventies **(Scheidegger, 1965**; **Shreve, 1967**; **Steinhaus, 1969)**.

Using Sharp method, there was no need to sample all points of catchments and analyze them, but one of the disadvantages of this method is that the factors of catchments do not play a role in determining the optimal points. In Sharp method, all primary basins have the same value and the values given to the intermediate basins have nothing to do with the parameters of the respective catchment and the characteristics of the upstream catchments. Sharp implemented the algorithm on a district of South Carolina with an area of approximately 1.9 square kilometers and 145 catchments, and finally 8 catchments were identified for final sampling.

Harmancioglu et al. presented an algorithm that took into account the properties of catchments for optimal sampling **(Harmancioglu et al., 2004)**. The basis of this algorithm was based on the method used by Lettenmaier et al. to detect contamination of the American water network, and they were able to reduce the number of samples from 81 to 47 by using this method **(Lettenmaier et al., 1984)**. By implementing this algorithm on Gadiz watershed in Turkey, Harmancioglu et al. were able to increase the number of watershed stations from 33 to 14. In this method, dynamic programming was used to find the optimal basins **(Harmancioglu et al., 2004)**. Dixon et al. proposed a method for

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determining the optimal sampling points using the simulated annealing algorithm. They implemented the algorithm on the Logan and Albert River areas in Queensland, Australia, and eventually introduced eight stations for optimal sampling **(Dixon et al., 1999)**.

In this paper for modeling the watershed network, using DEM images in the GIS, a graph model is presented and a matrix model related to the graph is implemented. In the next step, the information of catchments is considered as a matrix and by presenting a new algorithm which has been developed from Sharp theory and coding in MATLAB, the key catchments are identified. According to this presented algorithm, using assigned weights and matrix operations, the critical paths are introduced and the optimal points are selected for sampling. The purpose of this algorithm is to reduce the number of sampling points and adjust the cost of sample analysis.

2. Methods

In this section we describe the mathematical methodology and algorithm that have been used for watershed modeling and sample location design.

2.1. Mathematical graph modeling of watershed network

In this section, special graphs made based on the network of watersheds and matrix patterns are used to analyze them. Due to the extent of watersheds, which sometimes extend for kilometers, it is necessary that this area is modeled to be able to analyze them using mathematical models and the main goal is to select the optimal sampling points from thousands. First, the geographical information of the catchments needs to be modeled. In this regard, the experts developed a technical method which prepare digital elevation model (DEM) using satellite images, topography and geomorphology of the region and river network. Using satellite images, these areas are divided into partition of 20-30 meters and each area is considered as a point. Then, using the information received from the satellite, an average altitude is assigned to each area (**Carranza, 2009**). The grid is formed based on the height of the areas from the highest to the lowest area (**Figure 1**).

2000	2500	2600	2800
1800	1900	2200	2400
1600	2000	2200	2600

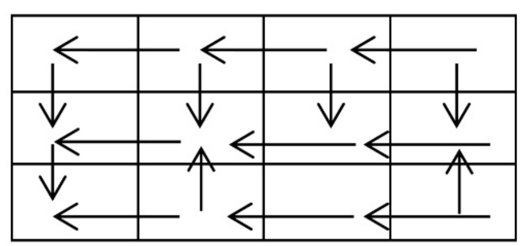

Figure 1: River network model based on the height of the areas obtained from DEM images

Sorting the catchment is carried out using the importance of its branches. In this method, the branches of the catchments are numbered in such a way that to each branch of the network, a number equal to the total number of upstream branches is assigned. In this way, the number 1 is assigned to the primary or input branches and the number 2 is assigned to the branch that consists of two first order branches. Suppose two small rivers with order m_1 and m_2 are connected at one point. It is obvious that the larger river originating from these two rivers has the order of $m_1 + m_2$ (**Sharp, 1971**). In **Figure 2**, the watershed network is sorted using the above method and as can be seen, the outlet branch is ranked as 16, which indicates the total number of sub-branches.

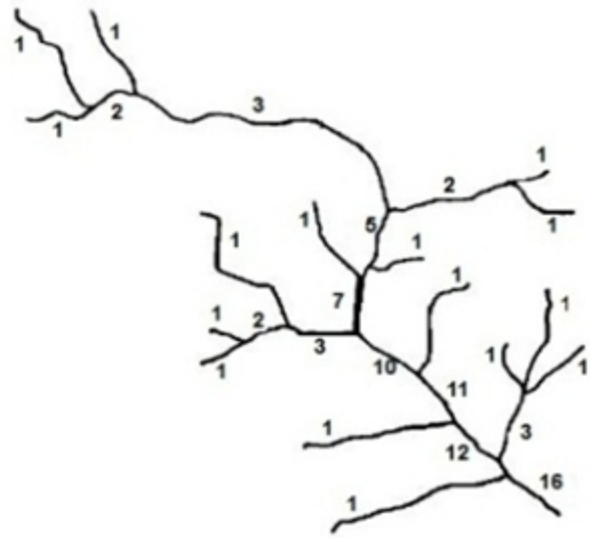

Figure 2: An example of watershed ranking (**Sharp, 1971**)

A river system can be supposed as a directed graph by considering a corresponding vertex for each branch of the watershed and determining the connection between the watershed network by directed edges in the graph. It is also possible to consider a vertex corresponding to each catchment, and based on which catchment water flows into another catchment, the connection between these two vertices is determined with a directed edge. As a result, the catchment network is modeled with a directed graph (**Figure 3**).

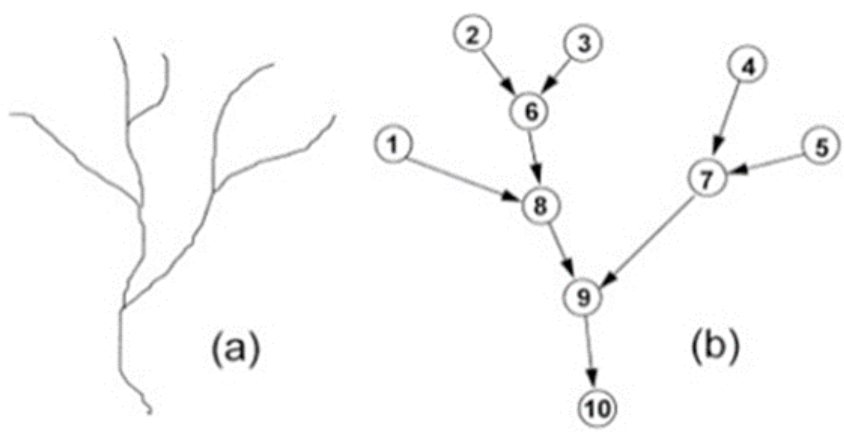

Figure 3: (a) A watershed network (b) Modeling by directed graph (**Dixon et al., 1999**)

Generally, every directed graph G can be modeled by a matrix. This matrix is called adjacency matrix. If we denote this matrix as A, in this matrix the entry a_{ij} is equal to 1 if there is a directed edge starting from the vertex v_i to the vertex v_j, otherwise it has a value of zero (**Harari, 1972**). As an example, a directed graph (**Figure 4**) is modeled as follows using its adjacency matrix (**Figure 5**).

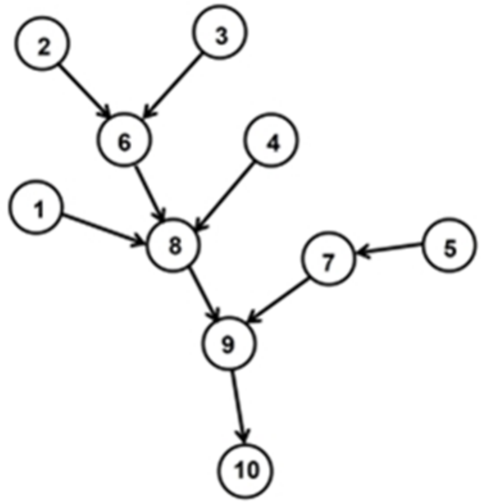

Figure 4: An example of directed graph modeled from the catchment network

$$
\boldsymbol{A} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Figure 5: Adjacency matrix of directed graph corresponding to **Figure 4**

The powers of adjacency matrix A are widely used in watershed network analysis. Suppose A is the adjacency matrix of the directed graph G with n vertices. In this case, the entry of the row i and the column j of the matrix A^m is equal to the number of paths of length m from the vertex v_i to the vertex v_j ($1 \le m \le n$) (**Harari, 1972**). Therefore, in the matrix A^2 corresponding to the graph in **Figure 4**, the entry a^2_{ij} is equal to 1 whenever water flows from the catchment i through an intermediate catchment to the catchment j, otherwise its value is zero. Also, in matrix A^3 related to this graph, the value a^3_{ij} is equal to 1 whenever water flows from catchment i through two intermediate catchments to catchment j otherwise its value is equal to zero.

According to the physical properties of watersheds, the graphs modeled on the catchment network are the union of several directed trees. Also in the matrix M, which is considered as the following **Equation 1**:

$$M = A + A^2 + A^3 + \ldots + A^n \qquad (1)$$

the elements of this matrix have the value of zero or one. It should be noted that the ones in row i represent the downstream catchments of catchment i and the ones in column j represent the upstream catchments of catchment j. Because each catchment flows into itself, the I + M matrix can be constructed to take this into account, which I is the identity matrix. Using the sum of the values in each column of this matrix, we can realize the importance of the catchment corresponding to that column. In addition, the column that has the most ones represents the outlet catchment. Also, columns with exactly one non-zero entry is known as inlet catchments. Due to the large number of the catchments, for calculation of I + M matrix, it can be simply used the following **Equation 2**:

$$I + M = (I - A)^{-1} \qquad (2)$$

where $(I-A)^{-1}$ is inverse matrix $(I-A)$. The correctness of the **Equation 2** can be easily proved by calculating $(I-A)(I+M)$ and vice versa and reaching to the identity matrix (I).

2.2. Algorithm and methodology

Due to the fact that the sampling of all watershed catchments is time consuming and costly task, there are a number of stations in each catchment that are the most important which need to be sampled. As a result, in order to access the optimal samples, unused stations should be eliminated in each catchment area and stations where the possibility of having an optimal sample is high should be considered. Suppose N_r is the total number of stations in the region and N_s is the number of stations that must be remained in the entire network. In this case, the number of selections for the stations that should be totally remained is calculated using **Equation 3** binomial coefficients as follows **(Harmancioglu et al., 2004)**:

$$C(N_r, N_s) = \binom{N_r}{N_s} = \frac{N_r!}{N_s!\,(N_r-N_s)!} \tag{3}$$

Where are:
N_r- the total number of stations in the region,
N_s- the number of stations that must be remained in the entire network.

Obviously, this number depends on the values of N_r and N_s and can be a large number of choices. Consider that the area is divided into k primary catchments. Suppose the number of stations in each catchment of i (i =1, 2, …, k) is equal to r_i and the number of stations remaining in that catchment is equal to s_i, therefore we have **Equation 4**:

$$N_r = \sum_{i=1}^{k} r_i\,,\;\; N_s = \sum_{i=1}^{k} s_i \tag{4}$$

Where are:
r_i- the number of stations in each catchment i,
s_i- the number of stations remaining in catchment i.

Because the number of stations that must be remained in the initial catchment i is not yet known, therefore s_i can select any of the values 0,1, . . ., r_i. If the number of selections for the total number of stations remaining in the area is denoted with the variable F and the number of selections for the total number of stations remaining in the catchment i is denoted with F_i, we have **Equations 5** and **6**:

$$F_i = \sum_{s_i=0}^{r_i} C(r_i, s_i) = 2^{r_i} - 1 \tag{5}$$

$$F = \prod_{i=1}^{k} F_i \tag{6}$$

As a result, F will have a very large value. Therefore, it is necessary to reduce the number of stations for sampling. In a sampling network, significant criteria must first be considered for selection of the optimum stations. These criteria should be used as a prioritized list at stations. In this way, the number of stations can be reduced by considering the priorities. For this purpose, for each initial catchment i, weight is allocated according to the existing criteria as follows (**Equation 7**):

$$W_i = \sum_{j=1}^{r_i} \sum_{l=1}^{m} w_{ijl} \tag{7}$$

Where are:
w_{ijl}- uniform data in the range (1,0) for property l in catchment i and for station j,
m- the number of properties that are considered as existing criteria.

In the Sharp method, each primary catchment is given the same weight. In this method, by determining the topological centre of the network, an optimal method for sampling of stream sediments can be achieved (**Sharp, 1971**). By definition, the topological centre of a watershed network is the branch that divide the network into two topological areas so that the number of outermost branches of two sides of the basin is equal. However, in our algorithm, the real value of the catchments is not the same and each branch does not have the same weight due to the topographical situation, length and slope of the branches, and the variety of geological units in the basin. Therefore, according to the weights assigned to the catchments, the first topological centre of the entire network can be calculated as follows (**Equation 8**):

$$w_{k1} = \frac{w_{ko}+1}{2} \tag{8}$$

Where are:
w_{k1}- *the first topological centre of the entire network for every basin k.*
w_{ko}- *the weight assigned to the outflow catchment in the basin k.*

Now suppose w_{s1} is the closest weight of the catchment to w_{k1} that exists in the basin network. Therefore, the relevant catchment is considered as a key catchment in the sampling priority after the outlet sample. The same steps are continued to find the catchments for the next optimal sampling and in step i in the remaining part, the topological centre is calculated as the following **Equation 9**:

$$w_{ki} = \frac{w_{ko}-\sum_{j=1}^{i-1} w_{sj}+1}{2} \tag{9}$$

Where are:
w_{ki}- *the i^{th} topological centre of the entire network for every basin k.*
w_{sj}- *the closest catchment weight to w_{kj}.*

This algorithm uses a combination of geographic information system (GIS), graph theory and the development of the Sharp method. Thus, a matrix model of watershed network has been prepared using GIS and graph theory. This matrix contains accumulated information and the location of samples in the catchment area, such as the catchment area and its physical and geomorphological properties, etc. This algorithm can use this matrix to provide the best combination of samples. In the proposed algorithm in this paper, at first the corresponding graph of the catchments is modeled using MATLAB software. Then, considering that each region has its own characteristics, the weight of catchments is determined based on the physical and geological characteristics of the catchments. Information about catchments is also stored in a matrix. By considering the output catchments and evaluating the upstream catchment areas and developing the Sharp method using the mentioned matrix and programming in MATLAB software, the optimal sampling points are determined. The implementation steps of this algorithm are presented in a flowchart as shown in **Figure 6**:

3. Case study

The study area is Khoy basin in West Azerbaijan in Iran. Khoy city is located in the south-eastern part of the region. This area on the basis of climatic situation has usually cold and semi-arid winters and mild summers and the average annual rainfall is 334 mm. The Aqchai and Aland rivers are the most important waterway systems in the region, which originate from the north-western highlands and the central mountainous regions, respectively, and flow into the Aras River. The valleys that appear in these areas are usually deep and their topographic slope in some areas exceeds 70 degrees. In this study, using geographical information system (GIS) and topography and slope of waterways, the region is divided into 457 catchments. According to the slope of the waterways and the geological condition of the region, the relationship between sub-basin catchments is determined. **Figure 7** shows the division map of the basins and the status of waterways in each catchment and the connections between these waterways in the study area.

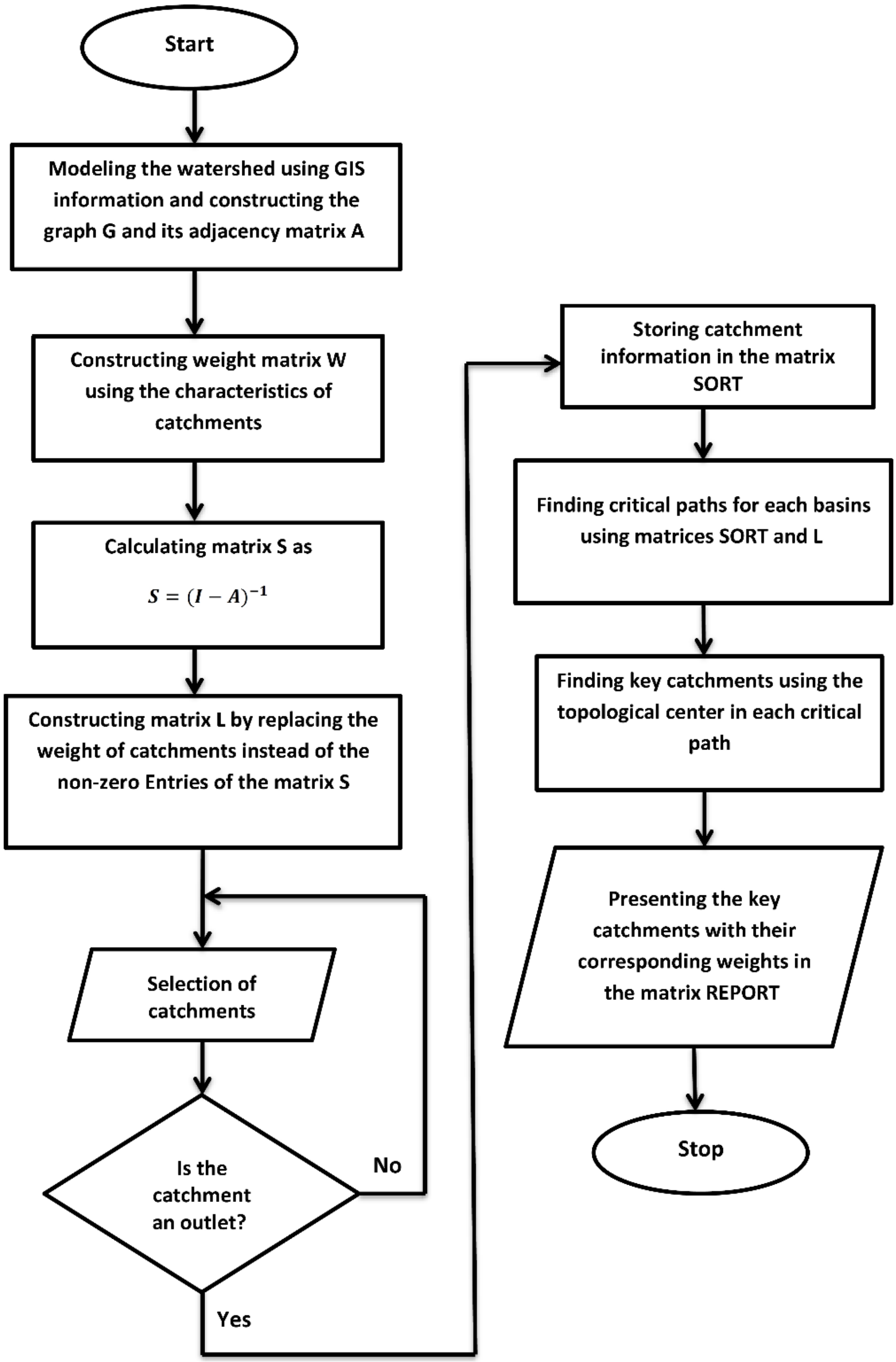

Figure 6: Flowchart presenting algorithm for finding key catchments

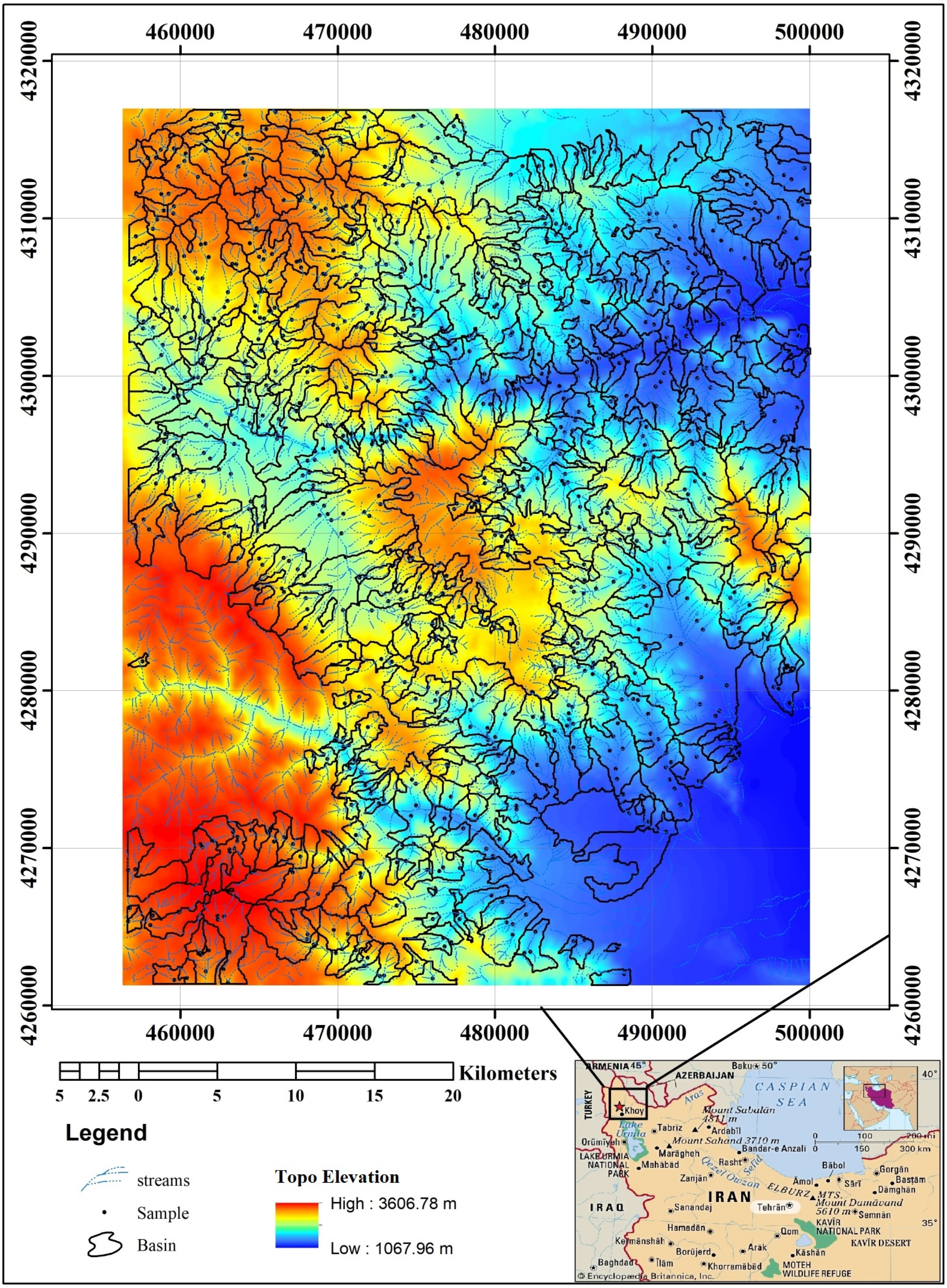

Figure 7: Condition of waterways, altitude changes and catchments of the region

In this section, at first one vertex corresponding to each catchment is considered and according to the connection status of upstream and downstream catchments, the directed graph and its adjacency matrix is formed. This matrix has 457 rows and 457 columns, which are shown in **Figure 8** as part of the adjacency matrix corresponding to the directed graph of the catchments, related to rows 111 to 133 and columns 116 to 140.

	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
111	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
112	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
113	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
114	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
115	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
117	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
118	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
119	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
121	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
122	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
123	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
124	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
125	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
126	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
127	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
128	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
129	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
131	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
132	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
133	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 8: A part of adjacency matrix

Using the adjacency matrix and MATLAB program, the directed graph of the catchments in the region was prepared. Part of this graph is presented in **Figure 9**. Using this graph, it is possible to easily identify the connections between the catchments and the upstream and downstream of a particular catchment.

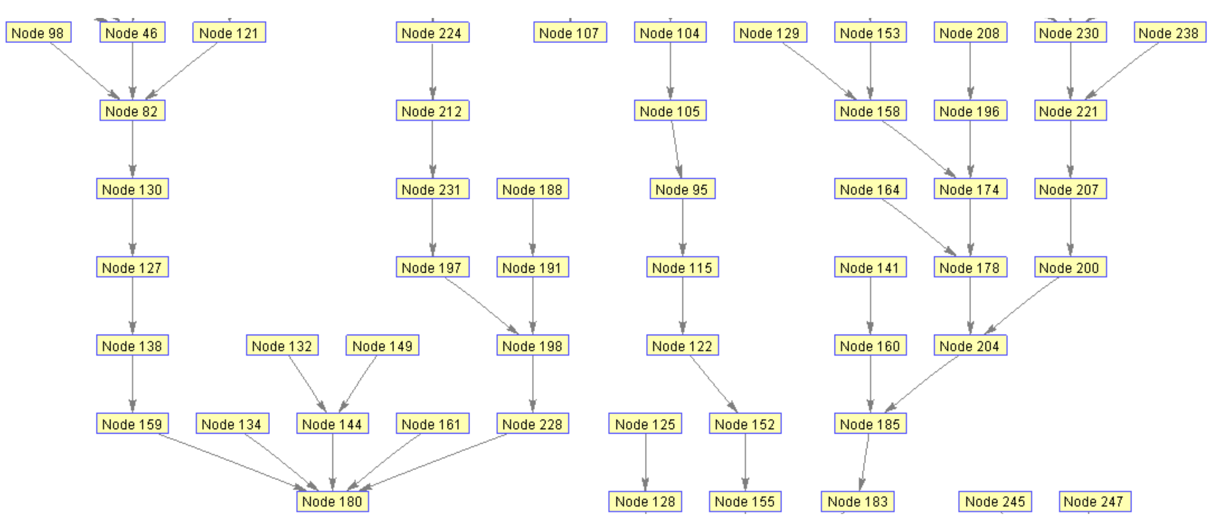

Figure 9: A part of directed graph corresponding to catchment situation

Using geographic information system (GIS), the desired parameters of each catchment including the area of the catchment, the total length and the number of waterways of each catchment, the number of rock units and the average slope of the catchment were extracted. The geological classes were considered in our study and one of the factors influencing to the weights is number of rock units in the catchments. These rock units of the district are very various, consist of sedimentary, partially metamorphic and in some parts ultrabasic to acidic volcanic and intrusive rocks. Due to the importance of the above parameters, a special weight was assigned to each of these indicators and as a result of combination of this information, a specific statistical weight was assigned to each of the catchments. This weight shows the importance of each catchment to be sampled. Table 1 shows a part of the weight matrix W.

Catchment No.	Weight	Catchment No.	Weight
1	2.72	26	2.71
2	2.03	27	1.44
3	2.55	28	2.32
4	3.32	29	3.40
5	3.42	30	1.82
6	3.50	31	3.17
7	2.51	32	2.51
8	2.22	33	1.42
9	1.28	34	4.31
10	2.43	35	2.32
11	1.63	36	3.20
12	2.62	37	1.86
13	2.08	38	3.28
14	3.51	39	2.00
15	2.86	40	3.87
16	2.70	41	1.80
17	1.77	42	1.99
18	1.64	43	2.96
19	3.09	44	1.92
20	1.73	45	2.94
21	2.54	46	3.78
22	2.42	47	3.65
23	2.16	48	2.51
24	2.83	49	2.89
25	3.01	50	3.95

Table 1: A part of weight matrix W assigned to the catchments

As explained in the flowchart (**Figure 6**), the matrix L was constructed using the weight matrix W and the mentioned steps of the algorithm. Using this matrix and MATLAB program, the outflow catchments and their corresponding numbers and their degree of importance are determined and the results were stored in the matrix SORT. Finally, using this matrix (SORT) and finding the critical path corresponding to each outflow catchment and applying **Equation 9**, the key catchments were identified (**Table 2** and **Figure 10**).

Catchment No.	Value	Catchment No.	Value	Catchment No.	Value
135	224.47	70	14.28	436	3.87
421	118.91	79	13.32	426	3.85
183	112.64	226	12.91	444	3.65
180	110	312	12.86	428	3.39
413	87.9	301	12.76	123	3.37
235	65.95	60	12.51	417	3.35
367	65.09	331	12.49	44	3.34
241	63.72	412	12.46	86	3.24
349	60.06	440	11.38	61	3.2
221	58.15	229	10.71	36	3.2
399	55.22	22	10.54	181	3.17
82	54.72	304	10.53	353	3.1
386	43.33	91	10.34	391	3.1
264	33.06	136	10.05	169	3
246	32.62	225	10.04	72	2.98
177	32.46	415	10.04	328	2.96
342	30.22	163	10.02	85	2.93
388	29.61	55	9.98	49	2.89
198	27.56	326	9.57	296	2.82
323	27.09	437	9.43	335	2.82
400	26.68	15	8.89	120	2.82
174	25.65	302	8.3	255	2.76
318	24.88	109	7.74	26	2.71
107	24.12	196	7.29	325	2.66
455	23.55	242	6.96	222	2.66
365	23.27	252	6.89	445	2.66
168	21.95	145	6.73	432	2.39
146	21.61	77	6.3	276	2.37
89	20.96	347	6.04	193	2.27
21	20.31	311	5.61	355	2.23
179	18.85	97	5.48	362	2.2
102	18.61	54	5.4	65	2.19
457	17.32	84	5.29	71	1.95
434	17.09	256	5.26	322	1.94
223	16.86	232	5.24	280	1.93
289	16.5	58	5.16	134	1.92
2	16.26	28	5.03	132	1.9
383	16.17	454	4.59	272	1.85
411	15.56	293	4.57	321	1.84
122	14.91	420	4.55	41	1.8
43	14.71	422	4.06	33	1.42
451	14.34	59	3.91		

Table 2: Selected Catchments for optimal priority sampling and their importance values

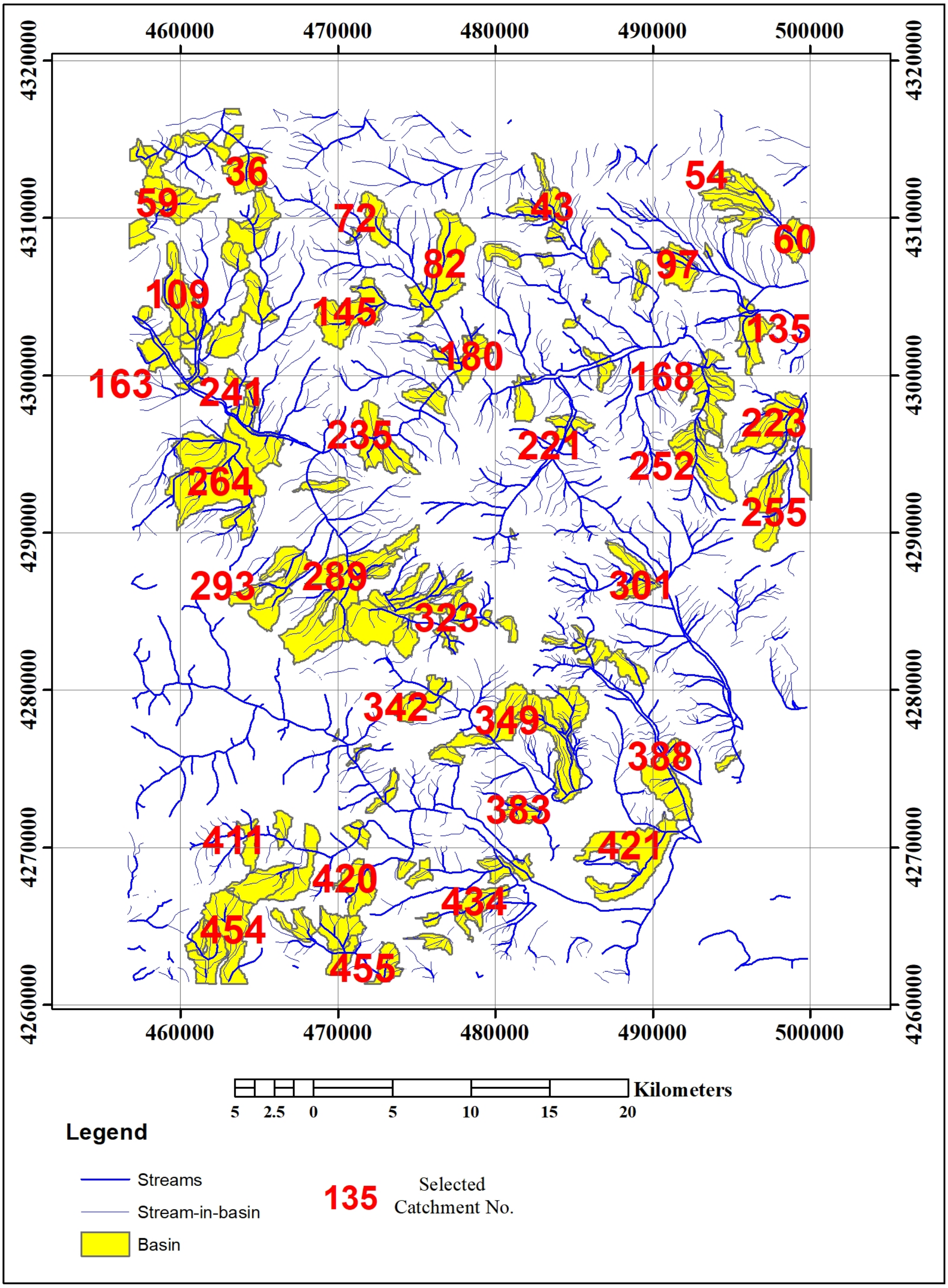

Figure 10: The location map of the optimal sample points (for better demonstration only some of the number of optimal catchments related to **Table 2** was shown in this figure)

4. Results

Using the proposed algorithm implemented on the Khoy region as a case study, the optimal catchments with their degree of importance for optimal sampling are specified and shown in **Table 2** and **Figure 10**. As can be seen in the table, the value of each catchment allows the sampler to be aware of its importance for the priority of the catchment sampling design. In this sampling system, sampling starts from the most important outlet with highest value and goes up towards the critical path of the relevant tree. In the second sampling priority, the outlet of the next high value tree is then sampled and thus the sampling work continues to reach to the source. Undoubtedly, the number and type of the rock units, catchment area, waterway length, catchments slope and so on are effective in tracking and determining the source of mineralization and must be considered in the designing of the sampling in order to achieve the goal with the minimum number of the samples. All of these important factors are considered in this algorithm.

Recent study shows that the use of this algorithm is able to reduce the number of required optimal samples to about one quarter. If the total number of catchments in the region is assumed to be 457, using a mathematical model and the algorithm presented in this research and finding critical paths, only 125 samples are required and this method is able to reduce the costs of sampling and analysis by one quarter.

5. Discussion and conclusion

Watershed sampling using traditional methods by taking the samples throughout area from all catchments and their subbranches is very expensive and time consuming. This research proposed a scientific methodology to optimize this sampling method. In this paper, the presented algorithm used a graph base watershed modelling that on the basis of GIS data implements weights on the main factors. These factors consist of area of the catchment, the total length and the number of waterways of each catchment, the number of rock units and the average slope of the catchment. We expect that this new algorithm can be used for precise modelling the watershed system and presenting the optimum points of sampling.

The proposed algorithm was implemented on a watershed network as a case study and the results showed that this algorithm is able to reduce the costs of sampling and analysis by one quarter.

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Sažetak

Modeliranje bazenske vododjelnice uporabom težinskih dijagrama te predstavljanje algoritma za odabir primjerenoga smjestišta uzorkovanja s ciljem smanjivanja troškova uzorkovanja i analiza

Uzorkovanje u području vododjelnice je najvažnija istraživačka metoda kod pridobivanja mineralnih sirovina. Ovdje je prikazano matematičko modeliranje vododjelnice s ciljem odgovarajućega uzorkovanja i optimiziranja broja uzoraka te smanjivanja cijene njihova uzimanja i analiziranja. Načinjeno je matematičko modeliranj odabranoga slivnoga područja uporabom DEM-a u GIS-u konstruiranjem izravnih grafova s otežavanjem u gornjem slivu. Zatim je načinjen matrični model kojemu su pridruženi težinski koeficijenti pomoću indeksa, uporabom koda u MATLAB-u. Time su opisani otjecajni dijelovi bazena te su određeni težinski koeficijenti za ta područja. Na kraju su izračunati kritični putovi toka pomoću simuliranih grafova te su za svaki takav tok određeni ključni bazeni glede uzorkovanja. Prikazani algoritam je provjeren na mreži vododjelnica u odabranom prostoru i rezultati su potvrdili kako je troškove uzorkovanja i analiza moguće smanjiti za četvrtinu.

Ključne riječi: vododjelnica; otežani grafovi; izravni grafovi; uzorkovanje; analiza

Author’s contributions

Both authors equally participate in all aspects of presented work.